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Surface quasi-Cherenkov free-electron laser

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The parametric quasi-Cherenkov free electron laser is studied in a surface diffraction geometry. It is shown that the application of the quasi-Cherenkov radiation mechanism makes it possible to realize the generation of wavelengths in the 0.1–10 Å region with a current density $j \sim 10^8$ A/cm^2, in spite of the strong requirements on the electron beam quality. The possibility of production of stimulated quasi-Cherenkov radiation in different spectral ranges is investigated.

1. Introduction

The problem of coherent X-ray generation by relativistic electron beams has aroused a steady interest [1]. Various types of coherent X-ray sources using Compton scattering of an electromagnetic wave by an electron beam, resonance transition radiation, channelled particle radiation etc. have been considered. Analysis of this problem showed that in order to realize an X-ray free electron laser on the basis of the mentioned mechanisms, a large beam current density $j > 10^{13}$ A/cm^2 is needed. However, it was shown in ref. [2] that induced X-rays may be produced by electron beams of much lower current density by using the parametric quasi-Cherenkov radiation mechanism. This spontaneous radiation mechanism, being the basis for producing induced radiation, was theoretically predicted and experimentally observed (see for example ref. [2]). The main peculiarity of quasi-Cherenkov radiation, in contrast with the ordinary one, is that its spectral and angular distributions are rather narrow and are determined mainly by the crystal parameters, i.e. $\delta\theta \sim \delta\omega/\omega \sim |\chi_0|^{1/2}$, where $\chi_0 = \epsilon_0 - 1$ and ϵ_0 is the crystal dielectric constant. Besides, the radiation generated by a relativistic particle beam can be observed not only at a small angle relative to the particle velocity but also at a large diffraction angle. The difference is the most prominent in the X-ray spectral region where the ordinary Cherenkov radiation is impossible on the whole and only the dynamical diffraction phenomenon creates the possibility of Cherenkov radiation because it changes the medium dielectric properties in the vicinity of the Bragg condition. That is why we speak of the parametric quasi-Cherenkov radiation.

A detailed analysis of the quasi-Cherenkov FEL is a crystal in given in refs. [3–5]. It was shown that, in the scheme of a solid state (crystal) FEL, the multiple scattering of the beam electrons by atoms destructs the synchronism condition between a particle and an emitted electromagnetic wave very fast. This leads, in its turn, to an essential increase of the generation threshold parameters. The reduction of the influence of multiple scattering is possible through the transition to a surface scheme for the FEL, where a particle beam moves over a periodic target at a distance $d \leq \lambda\gamma$ (λ is the photon wavelength, γ is the Lorentz factor) or at a small angle relative to this target $\psi \leq |\chi_0|^{1/2}$. These two cases have both common features and common differences. In both cases the distributed feedback is formed by surface dynamical diffraction of emitted photons but these cases are distinguished, for example, by particle trajectory parts which make the main contribution to the generation process. It is also important that in the first case a magnetic field, directed along the target surface, is needed because the spectrum of photon generation coincides with the spectrum of photon absorption by beam electrons in vacuum. Let us consider these two cases separately.

2. Generation threshold for quasi-Cherenkov radiation in the surface diffraction geometry

Let a relativistic electron (positron) beam of velocity u incident at an angle $\psi \sim \sqrt{|\chi_0|}$ with respect to the surface of a parallel plate. We consider a spatially periodic target with a length L ($0 < z < L$). It can be a

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crystal for X-ray radiation or a three-dimensional artificial structure with a period chosen in such a way that emitted photons satisfy the surface diffraction condition for a reciprocal lattice vector τ:

$$\tau = \frac{2\pi}{a}x + \frac{2\pi}{b}y + \frac{2\pi}{c}z,$$

where a, b, c are the periods along the x, y, z axes, respectively. The vector τ is assumed not to lie on the target surface but to be directed at a small angle $\psi_\tau \leq |\chi_0|^{1/2}$. Now we do not need two gratings to create the distributed feedback in the case of relativistic particle beam as it was suggested in ref. [6]. In the case of $|\chi_0| \ll 1$ the wave vectors of photons emitted by the electrons are also directed at angles $\sim|\chi_0|^{1/2}$ with respect to the surface of the plate.

The dispersion equation for quasi-Cherenkov radiation modes has the following form [3]:

$$\{k^2c^2 - \omega^2(\epsilon_0 - \mathrm{i}\Gamma)\}\{k_\tau^2c^2 - \omega^2\epsilon_0\} - \omega^4 r = 0. \quad (1)$$

In Eq. (1) $r = \chi_\tau\chi_{-\tau}$; $\epsilon_0 = 1 + \chi_0$; χ_0, χ_τ are the components of the series expansion of the crystal susceptibility versus the reciprocal lattice vector. The quantity Γ in Eq. (1) is determined as:

$$\Gamma = -\frac{\mathrm{i}}{\gamma}\left(\frac{\omega_\mathrm{L}}{\omega}\right)^2 \times\left(\frac{\boldsymbol{u}\cdot\boldsymbol{e}_\sigma}{c}\right)^2 \frac{k^2c^2 - \omega^2}{(\omega - \boldsymbol{k}\cdot\boldsymbol{u})^2 - \dfrac{\hbar^2(k^2c^2-\omega^2)^2}{4m^2c^4\gamma^2}}$$

in the limit of "cold" beam and

$$\Gamma = -\frac{\sqrt{\pi}}{\gamma}\left(\frac{\omega_\mathrm{L}}{\omega}\right)^2\left(\frac{\boldsymbol{u}\cdot\boldsymbol{e}_\sigma}{c}\right)^2\frac{k^2c^2-\omega^2}{\delta_0^2}x^\mathrm{t}\exp\{-(x^\mathrm{t})^2\}$$

in the limit of "hot" beam.

Here ω_L is the Langmuir frequency of an electron beam, $x^\mathrm{t} = (\omega - \boldsymbol{k}\cdot\boldsymbol{u})/(\sqrt{2}\delta_0)$; $\delta_0^2 = (k_1^2\Psi_1^2 + k_2^2\Psi_2^2 + k_3^2\Psi_3^3)u^2$; $\Psi_i = \Delta u_i/u$ is the velocity spread.

Eq. (1) has 6 solutions in the "cold" beam limit and 4 in the "hot" beam limit (k_{zi}, where $i = 1$ to n, $n = 6$ or $n = 4$). Therefore, the electromagnetic field can be represented in the form

$$\boldsymbol{E} = \exp\{\mathrm{i}\boldsymbol{k}_0\cdot\boldsymbol{r}\} + a\exp\{-\mathrm{i}\boldsymbol{k}_0^{(-)}\cdot\boldsymbol{r}\} + b\exp\{\mathrm{i}\boldsymbol{k}_{0\tau}^{(-)}\cdot\boldsymbol{r}\} \quad (2a)$$

before the target,

$$\boldsymbol{E} = \sum_{i=1}^{n} c_i\exp\{\mathrm{i}\boldsymbol{k}_i\cdot\boldsymbol{r}\}(1 + s_i\exp\{\mathrm{i}\tau\cdot\boldsymbol{r}\}) \quad (2b)$$

in the target,

$$\boldsymbol{E} = f\exp\{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}\} + g\exp\{\mathrm{i}\boldsymbol{k}_\tau\cdot\boldsymbol{r}\}, \quad (2c)$$

after the target, where $\boldsymbol{k} = (\boldsymbol{k}_\perp, k_{0z})$, $\boldsymbol{k}_\tau = \boldsymbol{k} + \tau$, $\boldsymbol{k}_0^{(-)}$ and $\boldsymbol{k}_{0\tau}^{(-)}$ correspond to the waves reflected from the target surface, $\boldsymbol{k}_0^{(-)} = (\boldsymbol{k}_\perp, -k_{0z})$, $\boldsymbol{k}_i = (\boldsymbol{k}_\perp, k_{zi})$.

Writing down the boundary condition for the electromagnetic field (2) and solving the obtained linear system we can derive the equation for the generation threshold.

$$G_\mathrm{thr} = \left(\frac{\gamma_\mathrm{rad}}{\psi}\right)^{2(m-1)}\frac{a_m}{(k|\chi_\tau|L_1)^{2(m-1)}L_1} + kb_m\chi_0'',$$

where m is the degeneration degree of the roots of the dispersion equation, $\gamma_\mathrm{rad} = (\boldsymbol{k}\cdot\boldsymbol{n})/k$, a_m and b_m are the constants dependent on the diffraction geometry, $L_1 = L/\psi$, $\boldsymbol{n}$ is the normal to the target surface and, for example, for the "cold' beam in the vicinity of double degenerate dispersion equation roots G_thr can be written in the following form:

$$G_\mathrm{thr} = \frac{k^2L_1^2}{4\gamma}\left(\frac{\omega_\mathrm{L}}{\omega}\right)^2\sin^2\phi\left(\chi_0' \pm |\chi_\tau|(-\beta)^{1/2} - \gamma^{-2}\right) \times\left(\chi_0' \pm |\chi_\tau|(-\beta)^{1/2}\right)f(x),$$

ϕ is the azimuthal angle, and

$$f(x) = [\sin(x)((2x + \pi n)\sin(x) - x(x + \pi n)\cos(x))]/\left[x^3(x + \pi n)^3\right]$$

is the spectral function.

If the condition $\gamma_\mathrm{rad}/\psi \ll 1$ is satisfied, the radiation losses through the target boundaries are reduced in comparison with the case of the ordinary volume diffraction because the lengths of interaction of a particle beam and photons with the target medium are $L_1 \sim L/\psi$ and $L_2 \sim L/\gamma_\mathrm{rad}$, respectively. Obviously, we can obtain $L_1 \ll L_2$ by varying the beam incidence angle. Besides, under surface diffraction the degree of dispersion root degeneration can increase and this will lead to a decrease of the electromagnetic wave group velocity and, consequently, to an increase of the time interaction between the emitted electromagnetic wave and the particle beam. The weakness of this situation is that the gain is proportional to L_1^2 for small L_1 and quickly falls with decreasing effective interaction length. As a result, the threshold beam current density is now about 10^{10} A/cm^2 for the X-ray region.

In the second case where a particle beam moves parallel to a periodic target surface at a distance $d \leq \gamma\lambda$, the radiation is formed along the whole particle trajectory in vacuum in the absence of multiple scattering. The spontaneous radiation for this case was considered in refs. [6,7].

As we mentioned above, to produce the induced radiation it is now necessary to introduce a magnetic field directed in the particle movement direction. By solving the corresponding boundary problem for a particle beam with a finite transverse size δ we can obtain the equation for threshold parameters.

The analysis of this equation shows that the most interesting situation will be observed in the case of a "cold" particle beam and a weak-absorbed target (i.e. $\omega''/\omega \gg \chi_0''$, where $\omega'' = \mathrm{Im}(\omega)$ and $\chi_0'' = \mathrm{Im}(\chi_0)$).

It is known that ω'' is the increment of the absolute instability and determines the growth rate over the threshold

$$\omega'' = \frac{1}{2}\left[\frac{3^{3/2}\omega\omega_L^2}{4\gamma^5}\left[1+\frac{kd_0}{2\gamma}\left(1+\frac{1-\beta}{\beta^2\alpha^2+4\beta r}\right)\right]^{-1} \times \exp\left(-\frac{2\omega d}{c\gamma}\right)\left[1-\exp\left(-\frac{2\omega\delta}{c\gamma}\right)\right]\right]^{1/3},$$

where $\beta = -k_z/k_{z\tau}$ is the diffraction asymmetry factor, $\alpha = (2k_z\tau_z + 2k_x\tau_x + \tau^2)/k^2$. The X-axis is chosen along the particle movement direction, the Z-axis is normal to the target surface, and d_0 is the width of the plate along the Z-axis.

Using the derived expression we can give some estimations for the growth rate over the threshold level: $\omega'' \sim 3\times10^{10}$ s^{-1} for $\gamma = 10$, $\omega = 3\times10^{15}$ s^{-1}, $j \simeq 10^4$ A/cm^2, $\epsilon^{1/2} = 1.56$, and $|\chi_\tau| \sim 10^{-2}$. It corresponds to the enhancement of the initial spontaneous radiation in $\exp(3\times10^{10}\ L_b/c)$ times, where L_b is the electron beam length.

3. Conclusion

The analysis of various versions of the parametric quasi-Cherenkov FEL shows that the most favourable situation takes place when the particle beam moves in vacuum over the target and distributed feedback is formed by surface dynamical diffraction of emitted photons. In this case we can realize the generation process, for example in the optical spectral range, with the help of particle beam already available and achieve rather high enhancement of the initial spontaneous radiation.

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