J. Phys.: Condens. Matter 3 (1991) 2421–2430. Printed in the UK

Diffraction x-ray radiation from a relativistic oscillator in a crystal

V G Baryshevsky and I Ya Dubovskaya
Institute of Nuclear Problems, Byelorussian State University, Minsk 220050, USSR

Received 30 October 1989, in final form 7 November 1990

Abstract. Diffraction of x-rays in radiation from a relativistic oscillator formed by an external ultrasonic (laser) wave in a crystal is considered. Ultrasonic (laser) excitation, in the general case, leads not only to the formation of a transverse oscillator but also to the formation of a set of lattices by which the emitted photons are diffracted. It is shown that, in the particular case of a transverse ultrasonic wave excitation at $\boldsymbol{\tau} \cdot \boldsymbol{b} \ll 1$, we can neglect the ultrasonic wave influence on diffraction process. Just this situation is considered in detail in the present paper. The expressions for the angular and spectral distributions of considered radiation are derived. It is shown that the radiation spectrum, in this case, is more complicated than that of ordinary channelling radiation and its intensity in the vicinity of the Bragg frequency can be even higher than the intensity of the latter.

1. Introduction

X-ray and γ-radiation of relativistic electrons (positrons) moving at a small angle relative to the crystallographic planes (axes) is analogous to the radiation of a relativistic one-dimensional (two-dimensional) oscillator with a frequency in the laboratory frame, determined by the difference between the energy zones ε_n, ε_f of transverse motion: $\Omega_{nf} = \varepsilon_n - \varepsilon_f$[1, 2]. The radiation frequency is evaluated from the Doppler effect (which is complex and anomalous in the general case).

This radiation, known in the literature as channelling radiation (see, e.g., [3]), experiences considerable changes under the conditions of diffraction of quanta, generated in a crystal. According to [4–6] in this case a new phenomenon occurs, which may be called the diffraction radiation of a relativistic oscillator, formed by channelled particles. A characteristic property of this radiation is that, unlike the usual radiation process of a relativistic particle, when quanta are emitted in the angular interval $\Delta\vartheta \sim 1/\gamma$ where γ is the particle Lorentz factor, the diffraction x-ray radiation is also observed at a large angle relative to the particle velocity; this results in the formation of a typical diffraction pattern.

A relativistic oscillator in a crystal not only may be formed as a result of radioactive transitions between zones of channelled particle transverse motion but also is observable during particle motion in a laser wave and when a channelled particle moves in a plane (axial) channel, bent by a variable external field (ultrasonic or laser wave), i.e. in some kind of an electrostatic undulator [7, 8]. In this case the oscillator frequency in the laboratory frame $\Omega' = \kappa_z u - \Omega$ where $\boldsymbol{\kappa}$ is the wavevector of an external wave in a

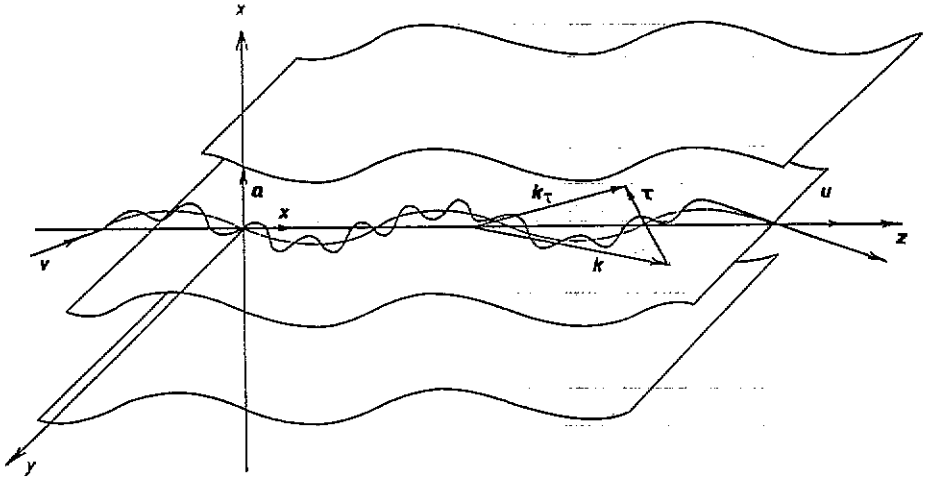

Figure 1. Qualitative illustration of electron motion and geometry of photon diffraction. The oscillation amplitude of medium atoms is parallel to the electron oscillation amplitude in ultrasonic wave ($\boldsymbol{b} \parallel \boldsymbol{a}$), $\boldsymbol{\tau}$ is placed near the x–z plane, as $(\boldsymbol{b} \cdot \boldsymbol{\tau}) \ll 1$.

crystal, Ω is its frequency (the z axis is chosen along the direction of an average particle velocity $\boldsymbol{u}$). The radiation frequency is determined by the Doppler effect: $\omega = \Omega'[1 - \beta_z n(\omega) \cos\vartheta]$ where $\beta_z = u/c$, $n(\omega)$ is the index of refraction at a frequency ω, ϑ is the radiation angle and c is the speed of light. Naturally, the possibility of a diffraction radiation phenomenon in itself does not depend on the mechanism of oscillator formation. The present paper gives the theory of the diffraction radiation from an oscillator formed by an external wave. The explicit formula determining the radiation intensity in the diffraction peak is obtained.

2. The trajectory of a channelled particle in a crystal being subjected to a variable external field

Thus, let a crystal in which a channelled particle (electron, positron, relativistic proton, etc) moves be subjected to a variable external field (electromagnetic or ultrasonic). Under the influence of this field, crystal nuclei begin to oscillate. As a result, the channel in which the particle is moving begins to bend, leading to the emergence of a variable force which causes oscillations of the particle. As a result, the particle moves in a dynamic undulator (figure 1) in the trajectory [8]

$$\boldsymbol{r}(t) = \boldsymbol{r}_{\mathrm{ch}}(t) + \boldsymbol{r}^{\mathrm{s}}(t)$$

where $\boldsymbol{r}_{\mathrm{ch}}(t)$ is the radius vector describing the ordinary high-frequency channelled particle motion and $\boldsymbol{r}^{\mathrm{s}}(t)$ is the radius vector describing the motion of particle in the dynamic undulator. Assuming that the frequency of the external variable field, e.g. an ultrasonic field, is much smaller than the frequency of particle oscillation in a crystal channel, we can consider these two kinds of particle motion independently: the ordinary channelled particle motion and the motion of the equilibrium trajectory centre of particle gravity inside the bent channel formed under the action of the external variable field. In this case, for a particle moving within a channel oriented along the z axis with an averaged velocity $\boldsymbol{u}$, we can write the trajectory inside the dynamic oscillator as

$$r^{s}_{\perp}(t) = a\cos(\Omega t - \boldsymbol{\kappa}\cdot\boldsymbol{u} + \delta) = a\cos(\Omega' t + \delta) \tag{1}$$

where $r^{s}_{\perp}(t)$ and $\boldsymbol{a}$ are the radius vector and amplitude of a particle oscillation in the x–y plane and δ is the initial oscillation phase in the external field. It should be noted that, if an ultrasonic wave amplitude satisfies the condition [8] $|\boldsymbol{a}| \ll uU/Ed\kappa^2$ (E is the particle energy, d is the crystal channel width and U is the depth of the potential well for a crystal channel), then the radius of the channel curvature due to the action of the ultrasonic wave is much larger than the radius of the trajectory curvature for the channelled particle incident on the crystal at the Lindhard angle. In this case the equilibrium trajectory of a positively charged particle gravity centre corresponds to the trajectory of a stable channelling regime, and the curvature of the crystal channel caused by the action of the ultrasonic wave leads only to the displacement Δ of the equilibrium trajectory centre of gravity during the particle movement through the crystal. That is why, for positively charged particles, for which $a_f + \Delta \leqslant d/2$ we can take into account the dechannelling effect because of channel curvature by considering the mean square angle of multiple scattering in this type of bent channel in the same way as in an amorphous medium [9, 10] (a_f is the amplitude of particle oscillation for the ordinary channelling regime).

The motion of the particle in the two independent trajectories leads to the appearance of two kinds of radiation from a relativistic particle: the component $r_{\mathrm{ch}}(t)$ results in the common channelling radiation and $r^{s}_{\perp}(t)$ leads to the radiation caused by the motion in the ultrasonic undulator correspondingly. The frequency of the quantum emitted in such an undulator is determined by the equation

$$\omega[1 - \beta_z n(\omega)\cos\vartheta] - \Omega' = 0.$$

If a given frequency is over the range of thousands of kiloelectronvolts, then emitted photons may experience diffraction by crystal planes. As a result, such a relativistic emitter causes the formation of a Bragg–Laue diffraction pattern as well as an ordinary x-ray emitter passing through a crystal plate with a radiation angular divergence $\Delta\vartheta \sim 1/\gamma$.

3. The dielectric constant of a crystal in the presence of external ultrasonic wave

In the case under consideration an essential difference arises compared with the diffraction radiation from an oscillator caused by a channelled particle. This is that atomic (nuclear) oscillations, resulting in the formation of an ultrasonic undulator, will simultaneously lead to the dielectric constant modulation in a crystal and, consequently, can change the diffraction process itself [11]. Indeed, the application of an external field (ultrasonic or laser wave) causes the electrons in the atoms and atomic centres of gravity to experience forced oscillations. In the same way as for ultrasonic waves, the centres of gravity of atoms, denoted $\boldsymbol{R}_m$, begin to oscillate. Inasmuch as the ultrasonic frequency is much smaller than typical oscillation frequencies of electrons in atoms, so the electrons adiabatically follow the motion of an atomic centre of gravity. In this case we may consider the motion of an atom as a whole. When a laser wave is applied, the situation is more complicated, because an electromagnetic wave directly affects not only nuclei but also electrons, causing them to experience forced oscillations. In this case the picture of the forced oscillations for the electrons in the inner shells of atoms, whose eigenfrequencies are much greater than that of a visible region, differs from the motion of electrons from the outer shells, whose oscillation frequencies may be smaller or close to an optical frequency.

In any case, however, the electron coordinate in an atom (the coordinate of the atom itself) in the presence of an external field may be written as

$$\boldsymbol{R}_m = \boldsymbol{R}_{0m} + \boldsymbol{b}\cos(\boldsymbol{\kappa}\cdot\boldsymbol{R}_{0m} - \Omega t + \delta_0) \tag{2}$$

where $\boldsymbol{R}_{0m}$ is the electron coordinate (the coordinate of the atomic centre of gravity) in the absence of an external field, $\boldsymbol{b}$ is the particle oscillation amplitude in the external field (for optical electrons or when light excites vibration transitions, $\boldsymbol{b}$ depends on Ω), and δ_0 is the initial oscillation phase.

As a result, the conductivity σ of the medium may be considered from a microscopic viewpoint as a sum of atomic conductivities, averaged over the crystal atomic state:

$$\sigma(\boldsymbol{r}; t, t') = \sum_{m=1}^{N} \sigma_m(\boldsymbol{r}; t, t') = \sum_{m=1}^{N} \sigma_m[\boldsymbol{r} - \boldsymbol{R}_m(t); t, t'] \tag{3}$$

where N is the number of atoms in a crystal and $\sigma(\boldsymbol{r}; t, t') = 0$ as $t' > t$.

It is well known that in the absence of an external field the conductivity $\sigma(\boldsymbol{r} - \boldsymbol{R}_m; t, t')$ may be written in the form

$$\sigma(\boldsymbol{r} - \boldsymbol{R}_{0m}; t, t') = \rho(\boldsymbol{r} - \boldsymbol{R}_{0m}) f(t - t').$$

Let us take in (3) the summation over the positions of atoms. For this purpose we shall introduce the Fourier transform of electron density $\rho(\boldsymbol{r})$ and write

$$\begin{aligned}\sigma(\boldsymbol{r}; t, t') &= \sum_m \rho(\boldsymbol{r} - \boldsymbol{R}_m(t)) f(t - t') = \frac{1}{(2\pi)^3}\sum_m \int \mathrm{d}\boldsymbol{q}\,\rho(\boldsymbol{q}) \\ &\quad \times \exp\{\mathrm{i}[\boldsymbol{r} - \boldsymbol{R}_{0m} - \boldsymbol{b}\cos(\boldsymbol{\kappa}\cdot\boldsymbol{R}_{0m} - \Omega t + \delta_0)\cdot\boldsymbol{q}]\} f(t - t') \\ &= \frac{1}{(2\pi)^3}\sum_m \int \mathrm{d}\boldsymbol{q}\,\rho(\boldsymbol{q}) \exp[\mathrm{i}\boldsymbol{q}\cdot(\boldsymbol{r} - \boldsymbol{R}_{0m})] \\ &\quad \times \sum_{n=-\infty}^{\infty} (-\mathrm{i})^n \mathcal{J}_n(\boldsymbol{q}\cdot\boldsymbol{b}) \exp[-\mathrm{i}n(\boldsymbol{\kappa}\cdot\boldsymbol{R}_{0m} - \Omega t + \delta_0)] f(t - t')\end{aligned}$$

where $\mathcal{J}_n$ is the Bessel function of nth order.

To take the summation over m we should note that

$$\sum_m \exp[-\mathrm{i}(\boldsymbol{q} + n\boldsymbol{\kappa})\cdot\boldsymbol{R}_{0m}] = \frac{(2\pi)^3}{V_0}\sum_{\boldsymbol{\tau}} \delta(\boldsymbol{q} + n\boldsymbol{\kappa} - \boldsymbol{\tau}) \tag{4}$$

where V_0 is the volume of a crystal lattice cell.

As a result, we obtain

$$\begin{aligned}\sigma(\boldsymbol{r}; t, t') &= \frac{1}{V_0}\sum_{\boldsymbol{\tau}}\sum_n \rho(\boldsymbol{\tau} - n\boldsymbol{\kappa}) \exp[\mathrm{i}(\boldsymbol{\tau} - n\boldsymbol{\kappa})\cdot\boldsymbol{r}] \\ &\quad \times (-\mathrm{i})^n \mathcal{J}_n((\boldsymbol{\tau} - n\boldsymbol{\kappa})\cdot\boldsymbol{b}) \exp[\mathrm{i}n(\Omega t - \delta_0)] f(t - t').\end{aligned}$$

Now we can carry out an explicit Fourier transformation in time for Maxwell's equations. We have

$$D(\boldsymbol{r},\omega)=E(\boldsymbol{r},\omega)+(4\pi\mathrm{i}/\omega)j(\boldsymbol{r},\omega)$$

$$j(\boldsymbol{r},\omega)=\frac{1}{V_0}\sum_{\tau}\sum_{n}\rho(\boldsymbol{\tau}-n\boldsymbol{\kappa})\exp[\mathrm{i}(\boldsymbol{\tau}-n\boldsymbol{\kappa})\cdot\boldsymbol{r}-\mathrm{i}n\delta_0]$$
$$\times(-\mathrm{i})^n\mathcal{J}_n((\boldsymbol{\tau}-n\boldsymbol{\kappa})\cdot\boldsymbol{b})f(\omega+n\Omega)E(\omega+n\Omega). \tag{5}$$

The process of scattering by an electron is much faster than the oscillation period $1/\Omega$ of an external excitation, i.e. the function $f(\omega)$ against ω has a much larger width than Ω. As a consequence, we may estimate $f(\omega+n\Omega)=f(\omega)$ with a high accuracy.

Let us now recall that, in the absence of an external field,

$$D(\boldsymbol{r},\omega)=\varepsilon_0(\boldsymbol{r},\omega)E(\boldsymbol{r},\omega)$$
$$\varepsilon_0(\boldsymbol{r},\omega)=1+\sum_{\tau}\chi_\tau\exp(\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{r}) \tag{6}$$

where $\varepsilon_0(\boldsymbol{r},\omega)$ is the dielectric constant of the medium, $\chi_\tau\sim\rho(\boldsymbol{\tau})$ is the Fourier transform of the crystal susceptibility. On comparison of (5) and (6), it should be noted that in the presence of an external field the relation

$$D(\boldsymbol{r},\omega)=E(\boldsymbol{r},\omega)+\sum_{\tau}\chi_\tau\exp(\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{r})E(\boldsymbol{r},\omega)$$

transforms into

$$D(\boldsymbol{r},\omega)=E(\boldsymbol{r},\omega)+\sum_{\tau,n}\chi_{n\tau}\exp[\mathrm{i}(\boldsymbol{\tau}-n\boldsymbol{\kappa})\cdot\boldsymbol{r}-\mathrm{i}n\delta_0]$$
$$\times(-\mathrm{i})^n\mathcal{J}_n((\boldsymbol{\tau}-n\boldsymbol{\kappa})\cdot\boldsymbol{b})E(\boldsymbol{r},\omega+n\Omega)$$

where $\chi_{\tau n}\sim\rho(\boldsymbol{\tau}-n\boldsymbol{\kappa})$. As a result, Maxwell's equations may be written as

$$-\nabla\times\nabla\times E(\boldsymbol{r},\omega)+\frac{\omega^2}{c^2}\bar{\varepsilon}_0(\boldsymbol{r},\omega)E(\boldsymbol{r},\omega)+\frac{\omega^2}{c^2}\sum_{\tau}\sum_{n\neq0}\chi_{\tau n}\exp[\mathrm{i}(\boldsymbol{\tau}-n\boldsymbol{\kappa})\cdot\boldsymbol{r}-\mathrm{i}n\delta_0]$$
$$\times(-\mathrm{i})^n\mathcal{J}_n((\boldsymbol{\tau}-n\boldsymbol{\kappa})\cdot\boldsymbol{b})E(\boldsymbol{r},\omega-n\Omega)=-\frac{4\pi\,\mathrm{i}\omega}{c^2}j_0(\boldsymbol{r},\omega) \tag{7}$$

where

$$\bar{\varepsilon}_0(\boldsymbol{r},\omega)=1+\sum_{\tau}\chi_\tau\mathcal{J}_0(\boldsymbol{\tau}\cdot\boldsymbol{b})\exp(\mathrm{i}\boldsymbol{\tau}\cdot\boldsymbol{r}).$$

Consecutive quantum-mechanical consideration leads to the analogous result for (7).

Thus, according to (7), the diffraction problem of radiation produced by a particle in the presence of an external field consists of the analysis of diffraction by a set of diffraction lattices. From (7) an important conclusion follows: if the atomic oscillation amplitude due to the action of an external field is parallel to the system of crystallographic planes relative to which diffraction is being studied or the condition $\boldsymbol{\tau}\cdot\boldsymbol{b}\ll1$ is fulfilled, then $\mathcal{J}_0\rightarrow1$, $\mathcal{J}_{n\neq0}\rightarrow0$ for a transverse ultrasonic wave ($\boldsymbol{b}\perp\boldsymbol{\kappa}$). This means that radiation diffraction is found to be the same as that due to a crystal in the absence of external excitation. Let us assume the above-stated condition to be satisfied and proceed to the consideration of the intensity of radiation generated by a particle moving in an ultrasonic undulator, formed in a crystal by external excitation. In other situations we should take into account the influence of the ultrasonic wave on the dynamic photon diffraction and we should use for the photon wavefunction the expressions from, for example, [11].

4. The spectral angular distribution of radiation

The spectral energy density of radiation per unit of solid angle, denoted $W_{n,\omega}$ ($\boldsymbol{n} = \boldsymbol{k}/|\boldsymbol{k}|$, where $\boldsymbol{k}$ and ω are the wavevector and frequency of an emitted quantum), the differential number of quanta, denoted $\mathrm{d}N_{n,\omega} = (1/\hbar\omega)W_{n,\omega}$, and also the polarization radiation parameters may be obtained, if we know the field $\boldsymbol{E}(\boldsymbol{r}, \omega)$ which gives a charge at a large distance from a crystal [12]:

$$W_{n,\omega} = (cr^2/4\pi^2)|\boldsymbol{E}(\boldsymbol{r}, \omega)|^2. \tag{8}$$

In order to determine the field $\boldsymbol{E}(\boldsymbol{r}, \omega)$, we should solve Maxwell's equations. The transverse solution may be found by means of a Green function of the equation satisfying an equation of the form

$$G = G_0 + G_0(\omega^2/4\pi^2c^2)(\varepsilon - 1)G$$

where G_0 is the transverse Green function of equation (7) with $\varepsilon = 1$; its explicit form is given, for example, in [13]. With the help of G we can readily find the field of interest to us:

$$E_i(\boldsymbol{r}, \omega) = \int G_{il}(\boldsymbol{r}, \boldsymbol{r}'; \omega) \frac{\mathrm{i}\omega}{c^2} j_l(\boldsymbol{r}')\, \mathrm{d}^3 r' \tag{9}$$

where $i, l = x, y, z$.

According to [14], at $r \to \infty$ the Green function is expressed in term of the solution $\boldsymbol{E}^{(-)}(\boldsymbol{r}, \omega)$ of homogeneous Maxwell's equations, which contains at infinity the incoming spherical wave:

$$\lim_{r\to\infty}[G_{il}(\boldsymbol{r}, \boldsymbol{r}'; \omega)] = \frac{\exp(\mathrm{i}kr)}{r} \sum_s e_i^s E_{kl}^{(-)s*}(\boldsymbol{r}', \omega)$$

where $\boldsymbol{e}^s$ is the unity polarization vector, $s = 1, 2$; $\boldsymbol{e}^\sigma \perp \boldsymbol{e}^\pi \perp \boldsymbol{k}$.

If a wave is incident on a crystal with a finite thickness, then, with $r \to \infty$,

$$\boldsymbol{E}_k^{(-)s}(\boldsymbol{r}, \omega) = \boldsymbol{e}^s \exp(\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}) + \text{constant} \exp(-\mathrm{i}kr)/r. \tag{10}$$

We can show that the solution $\boldsymbol{E}_k^{(-)s}$ is related to the solution $\boldsymbol{E}_k^{(+)s}$ of Maxwell's equations, describing a plane-wave scattering by a target (crystal), in the following way:

$$\boldsymbol{E}_k^{(-)s*} = \boldsymbol{E}_{-k}^{(+)s}.$$

With the help of (9),

$$E_i(\boldsymbol{r}, \omega) = \frac{\exp(\mathrm{i}kr)}{r} \frac{\mathrm{i}\omega}{c^2} \sum_s e_i^s \int \boldsymbol{E}_k^{(-)s*}(\boldsymbol{r}', \omega) \cdot \boldsymbol{j}_0(\boldsymbol{r}', \omega)\, \mathrm{d}^3 r'. \tag{11}$$

As a result, the spectral energy density of photons with polarization $\boldsymbol{e}^s$ may be written as

$$W_{n,\omega}^s = \frac{\omega^2}{4\pi^2c^2} \left| \int \boldsymbol{E}_k^{(-)s*}(\boldsymbol{r}, \omega) \cdot \boldsymbol{j}_0(\boldsymbol{r}, \omega)\, \mathrm{d}^3 r \right|^2 \tag{12}$$

$$\boldsymbol{j}_0(\boldsymbol{r}, \omega) = \int \exp(\mathrm{i}\omega t) \boldsymbol{j}_0(\boldsymbol{r}, t)\, \mathrm{d}t = Q \int \exp(\mathrm{i}\omega t) \boldsymbol{v}(t) \delta(\boldsymbol{r} - \boldsymbol{r}(t))\, \mathrm{d}t. \tag{13}$$

where Q is the particle charge and $v(t)$ is the particle velocity at a given moment. Let us substitute (13) into (12) and obtain

$$\mathrm{d}N^{s}_{n,\omega} = \frac{\omega Q^2}{4\pi^2 \hbar c^3} \left| \int_{t_0}^{t_1} E^{(-)s*}_{k}(r(t), \omega) \cdot v(t) \exp(\mathrm{i}\omega t)\, \mathrm{d}t \right|^2 . \tag{14}$$

Integration in (14) is carried out over the whole interval of particle motion. Further, we concentrate our attention on the radiation generated during the period of motion inside a crystal. If the thickness of a target is large in comparison with the vacuum coherent length of radiation, then the contribution to intensity from the radiation generated in front of the crystal plate (in vacuum behind the crystal plate) may be ignored [4, 5].

The solutions $E^{(-)s}_{k}$ which are necessary for explicit determination of $\mathrm{d}N^{s}_{n,\omega}$ have been found previously [5, 6] and, in the case of two-beam diffraction, they take the form (their expression is given for a required region inside a crystal and here we assume the fulfilment of geometrical conditions when we can omit the influence of the ultrasonic wave on photon diffraction by a crystal)

$$\begin{aligned} E^{(-)s*}_{k} = -e^{s} \exp(-\mathrm{i}k \cdot r)\{\zeta^{s}_{01} \exp[\mathrm{i}(\omega/c\gamma_0)\delta^{s}_{1}(L-z)] + \zeta^{s}_{02} \exp[\mathrm{i}(\omega/c\gamma_0)\delta^{s}_{2}(L-z)]\} \\ + e^{s}_{\tau}\beta_1 \exp(-\mathrm{i}k_{\tau} \cdot r)\{\zeta^{s}_{\tau 1} \exp[\mathrm{i}(\omega/c\gamma_{\tau})\delta^{s}_{1}(L-z)] \\ + \zeta^{s}_{\tau 2} \exp[\mathrm{i}(\omega/c\gamma_{\tau})\delta^{s}_{2}(L-z)]\} \end{aligned} \tag{15}$$

where

$$\zeta^{s}_{0(1,2)} = \mp(2\delta^{s}_{1,2} - g_0)/2(\delta^{s}_{2} - \delta^{s}_{1}) \qquad \zeta^{s}_{\tau(1,2)} = \mp g^{s}_{\tau}/2(\delta^{s}_{2} - \delta^{s}_{1}) \qquad k_{\tau} = k + \tau$$

$$\delta^{s}_{1,2} = \tfrac{1}{4}\{g_0(1+\beta_1) - \alpha\beta_1 \pm [(g_0(1-\beta_1) + \alpha\beta_1)^2 + 4\beta_1 g^{s}_{\tau}]^{1/2}\}$$

$$\beta_1 = \gamma_0/\gamma_{\tau} \qquad \gamma_0 = k_z/k \qquad \gamma_{\tau} = k_{\tau z}/k \qquad g_0 \equiv \chi_0 \qquad g^{s}_{\tau} = \chi^{s}_{\tau}\mathcal{J}_0(\tau \cdot b).$$

Let us substitute (15) into (14) and take into account the fact that, in the case under consideration, the particle trajectory and its velocity at the moment t are represented as

$$r(t) = ut + r^{s}_{\perp}(t) + r_{\mathrm{ch}}(t) = ut + a\cos(\Omega' t + \delta) + r_{\mathrm{ch}}(t)$$

$$v(t) = u + v^{s}_{\perp}(t) + v_{\mathrm{ch}}(t) = un_z - a\Omega' \sin(\Omega' t + \delta) + v_{\mathrm{ch}}(t)$$

where u is the constant component of a particle velocity. As a result, for the spectral angular distribution $\mathrm{d}N_{0n,\omega}$ of photons emitted in the direction of particle motion ($\mathrm{d}N_{\tau n\omega}$ in the direction of diffraction and the spectral angular distribution) in the case of Laue diffraction one may obtain

$$\begin{aligned} \mathrm{d}N^{s}_{0(\tau)n,\omega} = \frac{\omega Q^2 B}{4\pi^2\hbar c^3} \Big\langle \Big| \sum_{\mu} \int_0^T [e^{s}_{0(\tau)} \cdot u + e^{s}_{0(\tau)} \cdot v_{\mathrm{ch}}(t) - (e^{s}_{0(\tau)} \cdot a)\Omega' \sin(\Omega' t + \delta)] \\ \times \exp\{-\mathrm{i}[k_{0(\tau)} \cdot r_{\mathrm{ch}}(t) + k_{0(\tau)} \cdot ut - k_{0(\tau)} \cdot a\cos(\Omega' t + \delta)]\} \\ \times \zeta^{s}_{0(\tau)\mu} \exp\Big(\mathrm{i}\frac{\omega}{c\gamma_{0(\tau)}}\delta^{s}_{\mu}(L - ut) + \mathrm{i}\omega t\Big)\, \mathrm{d}t \Big|^2 \Big\rangle \end{aligned} \tag{16}$$

$$B = \begin{cases} 1 & \tau = 0 \\ \beta_1 & \tau \neq 0 \end{cases} \qquad k_0 \equiv k$$

where $\langle\,\rangle$ is the average over points of entrance into a channel (both $r_{\mathrm{ch}}(t)$ and $v_{\mathrm{ch}}(t)$ depend on entrance points), T is the time of passage of a particle through a crystal plate;

in the general case, $\boldsymbol{r}_{\mathrm{ch}}(t)$ is represented in the form $\boldsymbol{r}_{\mathrm{ch}}(t) = \Sigma_f \boldsymbol{a}_f \cos(\Omega_f t + \delta_f)$, where $\boldsymbol{a}_f$ is the particle oscillation amplitude in a straight crystal channel, Ω_f is the oscillation frequency in the laboratory frame and δ_f is the initial phase.

Let us expand the exponent and containing $\cos(\Omega' t + \delta)$ in (16) in terms of Bessel functions. As a consequence, the expression for radiation intensity (equation (16)) may be written as

$$\mathrm{d}N^{s}_{0(\tau)\boldsymbol{n},\omega} = \frac{\omega Q^2 B}{4\pi^2 \hbar c^3}\left\langle \left| \sum_{m=-\infty}^{\infty} \sum_{\mu} \int_0^T [\boldsymbol{e}^{s}_{0(\tau)} \cdot \boldsymbol{u} + \boldsymbol{e}^{s}_{0(\tau)} \cdot \boldsymbol{v}_{\mathrm{ch}}(t) - (\boldsymbol{e}^{s}_{0(\tau)} \cdot \boldsymbol{a})\Omega' \sin(\Omega' t + \delta)] \right.\right.$$
$$\times \exp\{-\mathrm{i}[m(\Omega' t + \delta) + \boldsymbol{k}_{0(\tau)} \cdot \boldsymbol{u}t + \boldsymbol{k}_{0(\tau)} \cdot \boldsymbol{r}_{\mathrm{ch}}(t)]\}$$
$$\left.\left. \times (-\mathrm{i})^m \mathcal{J}_m(\boldsymbol{k} \cdot \boldsymbol{a}) \zeta^{s}_{0(\tau)\mu} \exp\left(\mathrm{i}\frac{\omega}{c\gamma_{0(\tau)}} \delta^{s}_{\mu}(L - ut) + \mathrm{i}\omega t\right) \mathrm{d}t \right|^2 \right\rangle. \tag{17}$$

Because of the explicit expression $\boldsymbol{r}_{\mathrm{ch}}(t)$, all the time integrals appearing in (17) are in the explicit form. However, the final expression is rather awkward, but it is simplified if we consider the case when the frequencies of particle oscillation in a crystal and of forced oscillation caused by the external field differ considerably (we are interested, in particular, in the case $\Omega' < \Omega_f$). In this case the frequencies of quanta emitted in a given direction because of these two mechanisms will also differ considerably because of proportionality between the radiation frequency and oscillation frequency at a given angle. This allows one to consider different mechanisms of radiation separately. For this reason we shall concentrate our attention on the radiation mechanism of interest to us in this paper, which is due to particle oscillations under the influence of an external field (i.e. oscillations due to motion in a bent channel).

When the above is taken into account, (17) may be represented in the form

$$\mathrm{d}N^{s}_{\tau\boldsymbol{n},\omega} = (\mathrm{d}N^{s}_{\tau\boldsymbol{n},\omega})_{\mathrm{PXR}} + (\mathrm{d}N^{s}_{\tau\boldsymbol{n},\omega})_{\mathrm{DRO}} \tag{18}$$
$$(\mathrm{d}N^{s}_{\tau\boldsymbol{n},\omega})_{\mathrm{PXR}} = \frac{\omega Q^2 \beta_1^2}{4\pi^2 \hbar c^3} (\boldsymbol{e}^{s}_{\tau} \cdot \boldsymbol{u})^2 |\mathcal{J}_0(\boldsymbol{k}_\tau \cdot \boldsymbol{a})|^2 \left| \sum_{\mu} \zeta^{s}_{\tau\mu} \frac{\exp(\mathrm{i}q^{s}_{z\mu}L) - 1}{q^{s}_{z\mu}} \exp(-\mathrm{i}q^{s}_{z\mu}L) \right|^2$$
$$(\mathrm{d}N^{s}_{\tau\boldsymbol{n},\omega})_{\mathrm{DRO}} = \frac{\omega Q^2 \beta_1^2}{4\pi^2 \hbar c^3} \sum_{m\neq 0} [(\boldsymbol{e}^{s}_{\tau} \cdot \boldsymbol{a}) m\Omega' + (\boldsymbol{e}^{s}_{\tau} \cdot \boldsymbol{u})(\boldsymbol{k}_\tau \cdot \boldsymbol{a})]^2 \left(\frac{\mathcal{J}_m(\boldsymbol{k}_\tau \cdot \boldsymbol{a})}{(\boldsymbol{k}_\tau \cdot \boldsymbol{a})}\right)^2$$
$$\times \left| \sum_{\mu} \zeta^{s}_{\tau\mu} \frac{\exp(\mathrm{i}q^{sm}_{z\mu}L) - 1}{q^{sm}_{z\mu}} \exp(-\mathrm{i}q^{sm}_{z\mu}) \right|^2$$
$$q^{s}_{z\mu} = q^{sm=0}_{z\mu} \qquad q^{sm}_{z\mu} = (1/c)(\omega - k_{z\tau}u - (\omega\beta/\gamma_\tau)\delta^{s}_{\mu} - m\Omega').$$

The first summand corresponds to parametric x-ray radiation (PXR) [15, 16] whose intensity is compared with the parametric x-ray radiation in the absence of an external field but decreases by a factor of $|\mathcal{J}_0|^2$ owing to the transfer of radiation energy into harmonics with $m \neq 0$. The second summand corresponds to particle radiation generated by its oscillations under the influence of an external field (DRO). As stated above, contributions to (18) from radiation caused by particle oscillations in a stationary crystal channel are not included here.

5. The contributions of spectral and angular intensities of radiation to a diffraction peak

In accordance with (18) in both cases the radiation spectrum is determined by equation

$$\mathrm{Re}(q^{s\mu}_{zm}) = (\omega/c)[1 - \beta_z \cos\vartheta - (\beta_z/\gamma_\tau)\,\mathrm{Re}\,\delta^{s}_{\mu}] - m\Omega' = 0. \tag{19}$$

For sufficiently thick crystals we can integrate equation (18) by the help of the δ function, which can be obtained as a limit of equation (19). Consequently, we may derive the spectral or angular distribution of radiation in the ultrasonic undulator of a crystal. For this purpose it is necessary to consider radiation kinematics (equation (19)) in more detail. Inasmuch as equations (18) and (19) are analogous to the expressions for diffraction radiation from channelled particle [5, 6], so we can use the analysis, carried out in [16, 17], for the diffraction x-ray radiation in a crystal.

According to [16, 17], under diffraction conditions the radiation spectrum becomes very complicated. Each radiation branch, in the absence of diffraction (in this case $\varepsilon = 1 - \omega_L^2/\omega^2$, $\omega_L^2 = 4\pi n_0 e^2/m_e$, where n_0 is the atomic density in a crystal), is split, in turn, into two subbranches.

Thus, diffraction results in the excitation of an additional branch in the complex Doppler effect with a frequency close to the Bragg frequency and in the formation of a radiation non-transparency region in the angular distribution. It should be noted that the radiation frequency of an additional diffraction branch changes a little with the radiation angle. As a result, the angular range, in which $|\alpha| \leqslant |g_0|$, may considerably exceed the standard angular interval, characterizing the diffraction of an x-ray external monochromatic wave.

Integrating (18) over frequencies, we can obtain the angular radiation distribution, for example, for σ polarization:

$$\frac{dN_\tau^\sigma}{d\Omega} = \frac{Q^2\beta_1^2 L}{8\pi}\sum_\mu \frac{\omega_\mu^2}{m\Omega'}|\zeta_{\tau\mu}^\sigma(\omega_\mu^\sigma)|^2\left\{1 - \frac{\omega_\mu^2}{\gamma_\tau m\Omega'}\mathrm{Re}\left[\left(\frac{\partial\varepsilon}{\partial\omega}\right)_{\omega=\omega_\mu^\sigma}\right]\right\}^{-1}$$
$$\times\left(\beta_1\omega_\mu^\sigma \sin^2\vartheta\cos\varphi\,\frac{\tau_y\cos\varphi - \tau_x\sin\varphi}{\tau_\perp} + m\Omega'\frac{\tau_z\sin\vartheta\sin\varphi - \tau_y\cos\varphi}{\tau_\perp}\right)^2 \tag{20}$$

where we assume the external wave to have a linear polarization along the x axis; the frequency ω_μ^σ is derived from (19).

Analysis shows that in the case under consideration the maximum in the angular radiation distribution is reached not when the exact Bragg diffraction condition is fulfilled ($\alpha = 0$) but rather close to the angles, corresponding to the degeneration points of the Doppler effect ($\partial\varepsilon/\partial\omega = 0$). A characteristic property of the angular distribution near this critical point is that the radiation intensity depends on the thickness as a function of $L^{3/2}$. If a particle possesses energy such that $1 - \beta \gg (1/\gamma_\tau)\,\mathrm{Re}(\varepsilon_\mu^s)$, then the magnitude of the radiation frequency is almost independent of the dielectric characteristics of the medium. In this case we may assume that $\omega_\mu^s = \omega_B = \beta\tau^2/2\tau_z + \Omega' = \Omega'(1 - \beta_z\cos\vartheta)^{-1}$, where ω_B is the Bragg frequency, satisfying the condition $\alpha = 0$. As a result, the integration of (18) is simplified. By integrating (18) over a solid angle with the centre at the diffraction angle we obtain the following expression ($L < L_{abs}$, where L_{abs} is the absorption length):

$$dN_{\tau m}^s/d\omega = (Q^2\beta_1^2 L/8)a^2\Omega'^2 m^2[1 - 2(\omega/\omega_{max} + 4P_\tau^s(\omega/\omega_{max})^2]B_\tau^s(\omega) \tag{21}$$

where $\omega_{max} = 2\gamma^2\Omega'$ is the maximum radiation frequency, P_τ^s is the polarization factor ($P_\tau^\sigma = (\tau_x^2 + 3\tau_y^2)/4\tau_\perp^2$; $P_\tau^\pi = (3\tau_x^2 + \tau_y^2)/4\tau_\perp^2$) and $B_\tau^s(\omega) = \Sigma_\mu|\zeta_{\tau\mu}^s(\omega)|^2$ is the function characterizing radiation diffraction.

To estimate the integral number N_τ^s of photons emitted by a particle and contributing to the diffraction peak, let us use the sharpness of the diffraction function $B_\tau^s(\omega)$.

As a result, the integral number of photons emitted by a particle oscillating in a dynamic undulator and contributing to the diffraction peak at a large angle relative to the direction of particle motion takes the form

$$N^{s}_{\tau m} = [\pi Q^2\sqrt{\beta_1}L|g^{s}_{\tau}(\omega_{\rm B})|^2\omega_{\rm B}^2 m^2/16|\tau_z|] \times a^2\Omega'^2[1 - 2(\omega_{\rm B}/\omega_{\rm max}) + 4P^{s}_{\tau}(\omega_{\rm B}/\omega_{\rm max})^2]. \tag{22}$$

6. Conclusion

Let us compare (22) with the number of emitted quanta that contribute to the diffraction peak under conditions for the diffraction of channelling radiation. As is well known (see, for example, [6]), the intensity of diffraction radiation generated by a particle and channelled in a straight crystal channel is proportional, in contrast with (22), to $(a_f\Omega_f)^2$. As a result, the diffraction radiation from the particle moving in a channel and bent by external ultrasonic wave can have an intensity larger than the diffraction radiation from the ordinary channelled particle if $(am\Omega')^2 > (a_f\Omega_f)$. This inequality can be realized for a standard ultrasonic field source and, as shown by the estimations, the influence of this wave on dechannelling process can be ignored in this situation. In conclusion, it should be noted that, owing to the squared charge dependence of the radiation intensity, the process considered above will be the most effective for relativistic nuclei, for which the ordinary channelling radiation is suppressed.

References

[1] Baryshevsky V G and Dubovskaya I Ya 1976 *Proc. Conf. on Interaction of Charged Particles in Single Crystals* (Moscow: Moscow University)
[2] Baryshevsky V G and Dubovskaya I Ya 1976 *Dokl. Akad. Nauk.* **231** 1335
[3] Fridman A, Gover A, Kurizki G, Ruschin S and Yariv A 1988 *Rev. Mod. Phys.* **60** 471
[4] Baryshevsky V G and Dubovskaya I Ya 1977 *Phys. Status Solidi* **82** 403
[5] Baryshevsky V G, Grubich A O and Dubovskaya I Ya 1978 *Phys. Status Solidi* b **88** 351
[6] Baryshevsky V G and Dubovskaya I Ya 1983 *J. Phys. C: Solid State Phys.* **16** 3663
[7] Baryshevsky V G 1971 *Dokl. Akad. Byeloruss. SSR* **15** 306
[8] Baryshevsky V G, Grubich A O and Dubovskaya I Ya 1980 *Phys. Lett.* **77A** 61
[9] Baryshevsky V G 1989 *Nucl. Instrum. Methods* B **44** 266
[10] Samsonov V M and Khanzadeev A V 1989 *Preprint* 1479 Nuclear Physics Institute, Leningrad
[11] Entin I R 1979 *Zh. Eksp. Teor. Fiz.* **47** 214
[12] Landau L D and Lifshitz E M 1982 *Electrodynamics and Condensed Media* (Moscow: Nauka)
[13] Mors E M and Feshbach G 1958 *Method and Theoretical Physics* (Moscow: Nauka)
[14] Baryshevsky V G 1976 *Nuclear Optics of Polarized Media* (Minsk: Byelorussian University)
[15] Baryshevsky V G and Feranchuk I D 1983 *J. Physique* **44** 913
[16] Baryshevsky V G, Gradovsky O T and Dubovskaya I Ya 1987 *Izv. Akad. Nauk, Ser. Phys.-Math.* **6** 77
[17] Gradovsky O T 1988 *Phys. Lett.* **126A** 291