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Visible surface quasi-Cherenkov FEL

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Abstract

A parametric quasi-Cherenkov free electron laser with surface diffraction distributed feedback is studied when an electron beam moves over the target surface. The parameters of such a device using the Los Alamos Advanced Free Electron Laser (AFEL) linac are given.

1. Introduction

As shown in Refs. [1,2] induced radiation from a relativistic particle beam can be obtained in a wide spectral region on the basis of parametric (quasi-Cherenkov) radiation (parametric quasi-Cherenkov FEL). Both volume [1] and surface [3] versions of such X-ray FELs were considered. We pointed out that a reduction in the destructive action on the generation process by multiple scattering of the particle beam by atoms can be achieved by transiting to the surface scheme for the FEL, where a particle beam moves over a periodic target at a distance $d \leq \lambda\gamma$ (λ is the photon wavelength, γ is the Lorentz factor) or at a small angle relative to this target $\Psi \leq |\chi_0|^{1/2}$ ($\chi_0 = \epsilon_0 - 1$, ϵ_0 is the medium dielectric constant).

In this case radiation is formed along the whole particle trajectory in vacuum with no multiple scattering. However, even in the best situation the beam current density necessary for achieving the oscillation threshold level in the X-ray spectral region is very high. On the other hand, in the visible spectral region the system parameters considered are available and the analysis shows that a parametric surface visible quasi-Cherenkov FEL can be constructed on the basis of existing accelerators. The basic physical processes for a quasi-Cherenkov X-ray FEL could be modeled with a visible one if an optical grating with $|\chi_0| \ll 1$ were chosen.

This report is devoted to the consideration of a scheme of a parametric optical quasi-Cherenkov FEL with $\chi_0/\chi_\tau \gg 1$ (χ_τ is the Fourier component of the dielectric susceptibility of the periodic medium), determined by the depth of modulation of a holographic optical diffraction grating.

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2. Generation equation of the quasi-Cherenkov FEL

Let us consider the scheme of an FEL as shown in Fig. 1. h is the distance between a particle beam and a target surface, δ is the transverse size of a particle beam, the axis X is chosen along the direction of motion of the beam, the axis Z is a normal to the target surface, and d is the width of the dielectric target. a, b and c are mirrors for the accumulation of energy radiated during the macropulse of an accelerator particle beam. Distributed feedback is formed by dynamical diffraction from an optical holographic diffraction grating. The reciprocal lattice vector characterizing diffraction in this case ($\tau = (2\pi n_1/a, 2\pi n_2/b, 2\pi n_3/c)$) is directed at an angle relative to the particle velocity and to the target surface. The generated radiation frequency is determined by the diffraction process.

The electromagnetic field excited in the system shown in Fig. 1 can be represented in the form:

$$\boldsymbol{E} = a \exp(\mathrm{i}\boldsymbol{k}_{0\tau}\boldsymbol{r}) + b \exp(\mathrm{i}\boldsymbol{k}_0^{(-)}\boldsymbol{r}), \tag{1a}$$

$$\boldsymbol{E} = a \exp(\mathrm{i}\boldsymbol{k}_{0\tau}\boldsymbol{r}) + c \exp(\mathrm{i}\boldsymbol{k}_n\boldsymbol{r}) + d \exp(\mathrm{i}\boldsymbol{k}_n^{(-)}\boldsymbol{r}), \tag{1b}$$

$$\boldsymbol{E} = a \exp(\mathrm{i}\boldsymbol{k}_{0\tau}\boldsymbol{r}) + f \exp(\mathrm{i}\boldsymbol{k}_0\boldsymbol{r}) + g \exp(\mathrm{i}\boldsymbol{k}_0^{(-)}\boldsymbol{r}), \tag{1c}$$

$$\boldsymbol{E} = \sum_{i=1}^{n} c_i \exp(\mathrm{i}\boldsymbol{k}_i\boldsymbol{r})(1 + s_i \exp(\mathrm{i}\tau\boldsymbol{r})) + c_r \exp(\mathrm{i}\boldsymbol{k}^{(-)}\boldsymbol{r}) + d_r \exp(\mathrm{i}\boldsymbol{k}_\tau^{(-)}\boldsymbol{r}), \tag{1d}$$

$$\boldsymbol{E} = c_t \exp(\mathrm{i}\boldsymbol{k}_0\boldsymbol{r}) + d_t \exp(\mathrm{i}\boldsymbol{k}_\tau^{(-)}\boldsymbol{r}), \tag{1e}$$

where $\boldsymbol{k}_0 = (\boldsymbol{k}_\perp, k_{0z})$, $\boldsymbol{k}_{0\tau} = \boldsymbol{k}_0 + \tau$, $k_{0z} = (\omega^2/c^2 - k_\perp^2)^{1/2}\boldsymbol{k}_0^{(-)}$, $\boldsymbol{k}^{(-)}$, $\boldsymbol{k}_\tau^{(-)}$ and $\boldsymbol{k}_{0\tau}^{(-)}$ are related to the waves reflected from the target surface, while $\boldsymbol{k}_0^{(-)} = (\boldsymbol{k}_\perp, -k_{0z})$, $\boldsymbol{k}_{0\tau}^{(-)} = (\boldsymbol{k}_{0\perp} + \tau_\perp; -k_{z\tau})$, $k_{0z\tau} = (\omega^2/c^2 - (\boldsymbol{k}_\perp + \tau_\perp)^2)^{1/2}$. Obviously the waves reflected from the target

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boundary inside the target (this is the boundary between the regions (1d) and (1e) for the transmitted wave $\boldsymbol{k}$ and between the regions (1c) and (1d) for the diffracted wave $\boldsymbol{k}_\tau$) do not fulfill the diffraction condition for the grating. Therefore, for these waves we can assume that the dielectric constant is equal to the ordinary dielectric constant of the medium without diffraction. That is, $\boldsymbol{k}^{(-)} = (\boldsymbol{k}_\perp, -\boldsymbol{k}_z)$, $\boldsymbol{k}_\tau^{(-)} = (\boldsymbol{k}_\perp, -k_{z\tau})$, where $k_z = ((\omega^2/c^2)\epsilon_0 - k_\perp^2)^{1/2}$ $k_{z\tau} = ((\omega^2/c^2)\epsilon_0 - (\boldsymbol{k}_\perp + \tau)_\perp^2)^{1/2}$. $\boldsymbol{k}_n = (\boldsymbol{k}_\perp, k_{zn})$, $\boldsymbol{k}_n^{(-)} = (\boldsymbol{k}_\perp, -k_{zn})$, $k_{zn} = (\omega^2/c^2 - k_x^2)^{1/2}$ is the solution of the dispersion equation for electromagnetic radiation modes inside the particle beam moving in vacuum and in a magnetic field $\boldsymbol{H}$ directed along the target surface. This field is needed because the spectrum of photon emission coincides with the spectrum of photon absorption by an electron beam in vacuum. The dispersion equation determining these modes is:

$$k^2c^2 - \omega^2 - \mathrm{i}(k_x^2c^2 - \omega^2)\sigma = 0, \tag{2}$$

where

$$\sigma = -\frac{\omega_L^2}{\mathrm{i}\gamma^3(\omega - k_x U)^2}$$

for the limit of "cold" beam and

$$\sigma = -\frac{\sqrt{\pi}}{\gamma^3}\frac{\omega_L^2}{\delta_0^2}x^t \exp(x^t)^2$$

for the limit of "hot" beam.

Here ω_L is the Langmuir frequency of the electron beam, $x^t = (\omega - \boldsymbol{k}\boldsymbol{u})/(\sqrt{2}\,\delta_0)$; $\delta_0^2 = (k_1^2\Psi_1^2 + k_2^2\Psi_2^2 + k_3^2\Psi_3^3)u^2$; $\Psi_i = \Delta u_i/u$ is the velocity spread of the electrons in the beam relative to the x,y,z axes, respectively.

$\boldsymbol{k}_i = (\boldsymbol{k}_\perp, k_{zi})$ where k_{zi} are the solutions of the diffraction dispersion equation (see Ref. [3]), $i = 1, 2$ for the case of $|\chi_0| \gg |\chi_\tau|$. $\boldsymbol{k}_{i\tau} = \boldsymbol{k}_i + \tau$, a, b, c, d, f, g, c_i, s_i, c_τ, d_r, c_t, d_t are the amplitudes of waves excited in the system under consideration. We assume that the synchronism condition is fulfilled only for the wave with $\boldsymbol{k}$. Consequently, we can consider that the particle beam does not affect the diffracted wave with $\boldsymbol{k}_\tau^{(-)}$ and this wave is not refracted and not reflected by the particle beam. The reflection coefficient for the wave $\boldsymbol{k}_0$ is included in the magnitude of $f \neq 1$. We will assume below for simplicity that k_y and τ_y are equal to zero.

The boundary conditions for the electromagnetic field (1) are written as

$$z = -H,$$

$$b \exp(\mathrm{i}k_{0z}H) = c \exp(-\mathrm{i}k_{zn}H) + d \exp(\mathrm{i}k_{zn}H),$$

$$-b \exp(\mathrm{i}k_{0z}H) = (k_{0z}/k_{zn})(c \exp(-\mathrm{i}k_{zn}H) - d \exp(\mathrm{i}k_{zn}H));$$

$$z = -h_0,$$

$$c \exp(-\mathrm{i}k_{zn}h) + d \exp(\mathrm{i}k_{zn}h) = f \exp(-\mathrm{i}k_{0z}h) + g \exp(\mathrm{i}k_{0z}h),$$

$$(k_{0z}/k_{zn})[c \exp(-\mathrm{i}k_{zn}h) - d \exp(\mathrm{i}k_{zn}h)] = f \exp(-\mathrm{i}k_{0z}h) - g \exp(\mathrm{i}k_{0z}h);$$

$$z = 0,$$

$$f + g = c_1 + c_2 + c_\tau,$$

$$f - g = k_{zi}/\epsilon_0 k_{0z}[c_1 + c_2 - c_\tau],$$

$$a = s_1c_1 + s_2c_2 + d_\tau,$$

$$a = (k_{z\tau}/\epsilon_0 k_{0\tau})[s_1c_1 + s_2c_2 - d_\tau];$$

$$z = d_0,$$

$$c_1l_1 + c_2l_2 + c_\tau l_r = c_t \exp(\mathrm{i}k_{0z}d),$$

$$(k_z/\epsilon_0 k_{0z})[c_1l_1 + c_2l_2 - c_\tau l_r] = c_t \exp(\mathrm{i}k_{0z}d),$$

$$s_1c_1l_1 + s_2c_2l_2 + d_\tau l_r = d_t \exp(\mathrm{i}k_{0z}d),$$

$$(k_{z\tau}/\epsilon_0 k_{0z\tau})[s_1c_1l_1 + s_2c_2l_2 - d_r l_{r\tau}] = -d_t \exp(-\mathrm{i}k_{0z\tau}d); \tag{3}$$

where $l_i = \exp(\mathrm{i}k_{zi}d)$, $l_\tau = \exp(-\mathrm{i}k_z d)$, $l_{z\tau} = \exp(-\mathrm{i}k_{z\tau}d)$; in the spirit of the present approximation we can assume $k_{zi} \simeq k_z$, if a multiplier of the wave amplitude.

Solving the above linear system of equations we can derive the generation (dispersion) equation in the following form:

$$I_n = (1 + \alpha_D A)/(A + \alpha_D), \tag{4}$$

where

$$\alpha_D = \left(k_{0z} - \frac{k_z}{\epsilon_0}\right)\left(k_{0z} + \frac{k_z}{\epsilon_0}\right)^{-1},$$

$$\alpha_D^\tau = \left(k_{0z\tau} - \frac{k_{z\tau}}{\epsilon_0}\right)\left(k_{0z\tau} + \frac{k_{z\tau}}{\epsilon_0}\right)^{-1},$$

$$A = \alpha_D \exp(\mathrm{i}k_z d)\frac{s_2(l_2 - \alpha_{D\tau}^2 l_\tau)l_1 - s_1(l_1 - \alpha_{D\tau}^2 l_\tau)l_2}{s_1(l_1 - \alpha_{D\tau}l_\tau) - s_2(l_2 - \alpha_{D\tau}^2 l_\tau)},$$

$$I_n = \kappa_0^2 \exp(-2\kappa_0 h)\frac{\omega_L^2}{\gamma^3(\omega - k_x U)^2}f(k, \delta)$$

for the limit of "cold" beam,

$$I_n = -\kappa_0^2 \exp(-2\kappa_0 h)\frac{\mathrm{i}\sqrt{\pi}\,\omega_L^2}{\gamma^3\delta_0^2}x^t \times \exp\left(-(x^t)^2\right)f(k, \delta)$$

for the limit of "hot" beam,

$$f(k, \delta) = \frac{\sinh(\kappa_n\delta)}{(\kappa_0^2 + \kappa_n^2)\sinh(\kappa_n\delta) + 2\kappa_0\kappa_n\cosh(\kappa_n\delta)},$$

$$\kappa_0 = k_{0z}/\mathrm{i}, \quad \kappa_{zn} = k_{zn}/\mathrm{i}.$$

The exact value of the frequency excited in the FEL is determined by the real part of the solution of Eq. (4) $\omega = \omega(k_x)$. Obviously, this frequency should satisfy both the synchronism condition and the diffraction condition for a given diffraction geometry characterized by the reciprocal lattice vector τ. From Eq. (4) one can obtain for the magnitude characterizing the growth rate of the electromagnetic field over the threshold level:

$$\omega'' = \omega\left\{1 + \frac{kd}{2\gamma}\left[1 + \frac{1-\beta}{\beta^2\alpha^2 + 4\beta r}\right]\right\}^{-1} \times \left\{\frac{I''_n}{\gamma^2} - \frac{k\chi''_0 d}{2\gamma\varepsilon_0}\left[1 + \frac{(1-\beta)r - \beta\alpha r''/(2\chi''_0)}{\beta^2\alpha^2 + 4\beta r}\right]\right\}. \quad (5)$$

The analysis shows that in the limit of a "hot" beam the process of generating coherent radiation is characterized by a threshold level while in the "cold" beam case the instability has no threshold behavior.

Using these results we consider as an example the possibility of constructing a surface, quasi-Cherenkov FEL in the visible spectral range ($\lambda \simeq 600$ nm) on the basis of the Los Alamos accelerator (parameters given in Ref. [4]). The analysis shows that the electron beam [4], with a transverse size less than 1 mm, should be considered "hot". The surface electromagnetic wave interaction length can be estimated as $\lambda\gamma \simeq 12$ μm for $\gamma = 20$ and $\lambda \simeq 600$ nm. A transverse size of the beam larger than this leads to a reduction in the efficiency of the interaction. A beam size of $\delta \simeq 1$ mm corresponds to a velocity spread of $\Delta\Psi_z \simeq 0.2$ mrad. This means that after passing a target with length $L = 0.5$ cm, the transverse displacement of an electron inside the beam is about 1 μm, less than the radiation interaction length, and much less than the transverse beam size. Using the parameters from Eq. (4), we can estimate the growth rate for a solitary microbunch of the beam ($\tau = 10$ s, $l_b = 0.3$ cm). The estimate, made in the linear approximation according Eq. (5), shows that the growth rate is $\simeq 0.2$ at $\omega'' \simeq 2 \times 10^{10}$/s. This means that the electromagnetic field enhancement is of the order of exp(400) for a 10 μs macrobunch containing 2×10^3 microbunches. Obviously, in this case we need to use a system of mirrors to accumulate electromagnetic radiation emitted by different microbunches of the beam. The distance between the mirrors should be chosen in such a way that the time of electromagnetic wave passing between the mirrors is equal to the interval between the two neighboring microbunches (for $\tau_{ij} \simeq 4.8$ ns, $L_{abc} \simeq 144$ cm).

3. Conclusion

The scheme considered here is, at first sight, very similar to the Smith–Purcell FEL considered in Refs. [5–8]. However, there are differences. First, we do not need two gratings or mirrors to provide feedback. In our case distributed feedback is formed by the dynamical diffraction which determines the excited radiation modes. This means that oscillation is possible without any mirrors. The two mirrors shown in Fig. 1 are necessary only to accumulate radiated energy from different microbunches of the accelerator beam because of the small length of the microbunches. They do not influence the radiation modes. In addition, the generated frequency does not depend

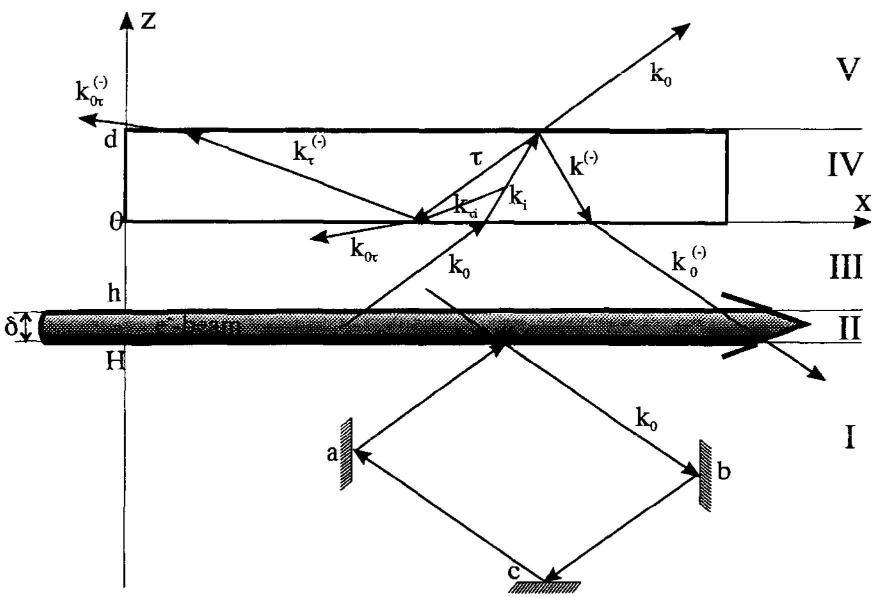

Fig. 1. Diagram of the quasi-Cherenkov surface FEL.

significantly on the particle energy in contrast with the Smith–Purcell FEL. A comparison between Cherenkov and diffraction (Smith–Purcell) radiation was made in Ref. [9]. The present study shows that the suggested scheme forms the basis for possible construction of a compact FEL in the visible spectral range.

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