GENERATION OF RADIATION IN VOLUME FREE ELECTRON LASERS AND PROBLEMS OF MATHEMATICAL MODELLING OF NONLINEAR PROCESSES IN SUCH GENERATORS

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Abstract

Paper presented is devoted to survey of progress in theoretical, mathematical and experimental investigations of Volume Free Electron Lasers.

1 Introduction

The conversion of kinetic energy of an electron beam into radiation underlies the operation of many electronic devices. Travelling wave tubes (TWT), backward wave tubes (BWT), free-electron lasers (FEL) operate on this principle. Nowadays FELs cover wavelength range from the microwave to ultraviolet [1]. FELs lasing in the ultraviolet range operate on the basis of self-amplified spontaneous emission (SASE) mechanism [2].

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Despite the success achieved in this area, there are many problems to be solved. One of them is that the generation frequency cannot be tuned in a wide spectral range. As a rule, high-efficient lasers have optimal, preliminarily designed parameters (such as electron beam characteristics, waveguide and resonator sizes, undulator and grating periods, modulation, etc.). Variations in any of these parameters, required for frequency tuning, sharply reduce the lasing efficiency. Another problem arises for large power lasing in microwave. In the generation of pulses with high power output, it is desirable that the cross section of the electron beam be large in order to reduce the current density occurring at a high total current. However, when the linear sizes of the cross-section exceed the generated wavelength, a large number of parasitic modes are excited in the resonator (waveguide). This leads to destructive interference and, as a consequence, to sharp decrease in the efficiency of generation.

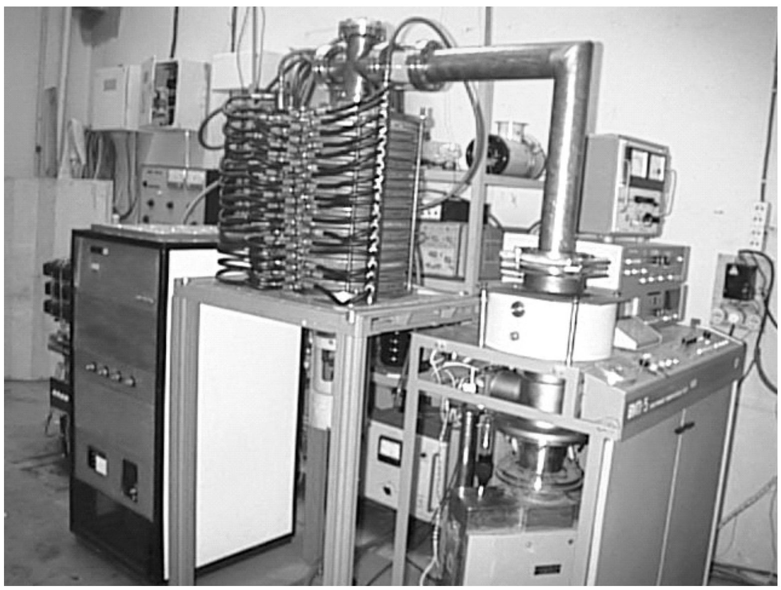


Figure 1: Installation VFEL-10 keV

One possible way to overcome these difficulties is to use volume distributed feedback (VDFB) in volume free electron laser (VFEL). VFEL operation based on mechanism of multi-wave VDFB was proposed in [3]-[4], theoretically developed in [5]-[9] and first lasing in mm wavelength range was obtained in 2001 by the INP group [10].

A major advantage of VFEL is that it provides mode discrimination

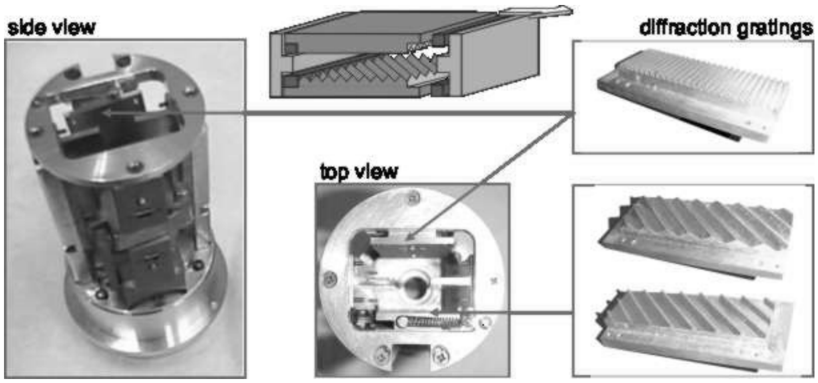


Figure 2: View of VFEL resonator

in an oversized system (when linear sizes of resonator cross-section exceed generated wavelength). Varying the VDFB geometry or the electron beam position relative to VDFB structure tune generated frequency. Besides VFEL gives possibility to reduce starting currents.

First experimental setup with electron beam energy up to 10 keV (VFEL-10) is depicted in Fig.1. The electron beam current of this experimental setup was ≤ 1 A. Radiated wavelength was in spectral range 4 - 6 mm. VFEL resonator of this setup is represented in Fig.2. It is formed by two diffraction gratings with different periods and two smooth side walls. The interaction of the first diffraction grating (exciting grating) with the electron beam leads to diffraction radiation. The second (resonant) grating provides distributed feedback of generated radiation with electron beam by means of Bragg dynamical diffraction. Resonator design allows to vary its parameters during the experiment. Exciting grating can move to change the gaps between gratings and between exciting grating and electron beam. Resonant grating can rotate to change orientation of grating grooves with respect to electron beam velocity that provides possibility to tune conditions of two-wave diffraction. The length of resonator is 100 mm, periods of the exciting grating and resonant grating in some experiment were 0.67 mm and 3 mm, respectively.

The next step in VFEL experimental study used pulsed electron beam source ($\tau_{pulse} \sim 200$ ns and $I \sim 1$–2 kA) with electron energy up to 250 keV (VFEL-250) (Fig.3) [13], [14]. Laser generation at wavelength ~ 3 cm was observed. "Grid" volume resonator (Fig.4) is used in these experiments [15]. It consists of metallic threads placed in rectangular waveguide

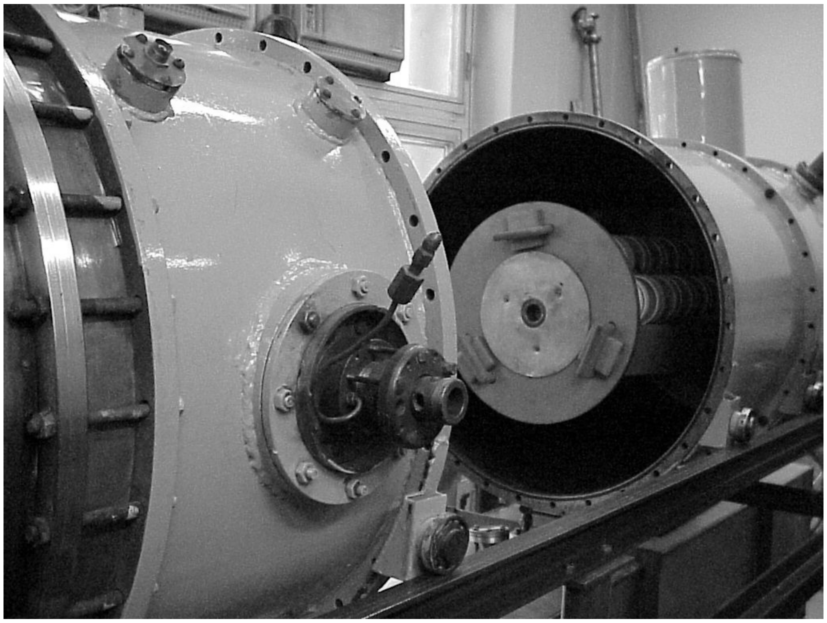


Figure 3: Installation VFEL-250 keV

through which electron beam is moving. Generation of electron beam in resonator with dielectric threads was studied theoretically and experiments with volume diffraction grating were performed in cold regime [16].

Linear regime of VFEL operation was investigated in [5]-[9] and other works. Duration of this stage is defined by the parameters of electron beam and resonator and usually is about 10^{-8}–10^{-9} s or less. Most part of energy is extracted from electron beam at nonlinear stage. Analysis of this stage requires severe numerical simulation which based on a system of multidimensional integro-differential equations. Boundary conditions can be written at different boundaries. Therefore we have to use numerical methods for solving such type of problem. Mathematical modelling of different types of VFELs was performed in [19]–[27].

2 VFEL basic principles

Simple schemes of two- and three-wave VFEL are presented in Fig.5. An electron beam with initial velocity $\mathbf{u}$ and current density j passes through the target of the length L which is a three-dimensional spatially-periodic

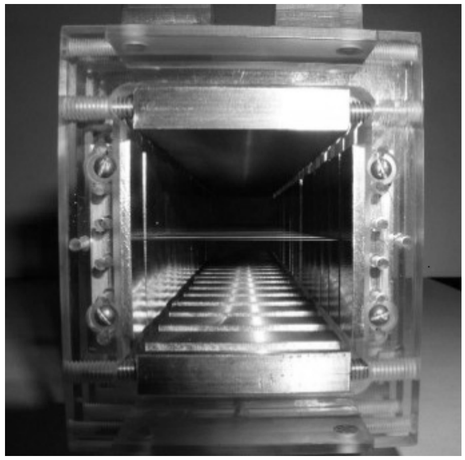
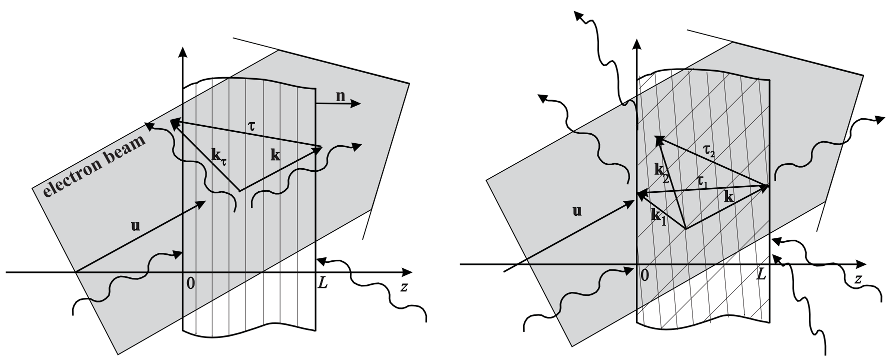


Figure 4: "Grid" volume resonator

structure. Under Bragg diffraction conditions [17] some strong coupled waves are generated. Under Cherenkov condition electrons bunch in a deceleration phase and emits coherently. In the case of amplification regime external electromagnetic waves are incident to the target. Oscillator regime can be realized without external waves. Insertion of external mirrors in oscillator regime can reduce starting currents and allows to realize generation in more optimal domains of parameters [18].



a b

Figure 5: Two-wave VFEL in Bragg geometry (a) and three-wave VFEL in Bragg-Bragg geometry (b)

There are two possible geometries in two-wave VFEL. Bragg geometry is represented in Fig.5a when waves with wave vectors $\mathbf{k}$ and $\mathbf{k}_\tau$ are directed in opposite directions relative to to surface normal vector. In other words:

$$(\mathbf{k},\mathbf{n}) \times (\mathbf{k}_\tau,\mathbf{n}) < 0$$

where $\mathbf{n}$ is a surface normal vector.

Other possible geometry is so-called Laue geometry, where $(\mathbf{k},\mathbf{n}) > 0, \quad (\mathbf{k}_\tau,\mathbf{n}) > 0$. The regime of amplification of external waves can be realized in this geometry. Without mirrors and external waves SASE regime can be realized too. It requires larger interaction length for shot noise can rise.

In three-wave case Fig.5b additional parameters exist which give possibility to adjust generation to more optimal region with reduced absorption. Besides three-wave VDFB can be realized in region of three root degeneration. In this case all three diffraction modes are in synchronism with electron beam and interaction occurs more intensively. There are three possible geometries in such a system. For Bragg-Bragg geometry (Fig.5b) the following inequalities take place:

$$(\mathbf{k}_0,\mathbf{n}) > 0, \quad (\mathbf{k}_1,\mathbf{n}) < 0, \quad (\mathbf{k}_2,\mathbf{n}) < 0,$$

Laue-Laue geometry is realized when $(\mathbf{k}_1,\mathbf{n}) > 0$, $(\mathbf{k}_2,\mathbf{n}) > 0$, $(\mathbf{k}_3,\mathbf{n}) > 0$. Bragg-Laue geometry is the case when waves are oriented so that $(\mathbf{k}_1,\mathbf{n}) > 0$, $(\mathbf{k}_2,\mathbf{n}) < 0$, $(\mathbf{k}_3,\mathbf{n}) > 0$.

Other possible type of VFEL is so-called surface quasi-Cherenkov VFEL. Such scheme was proposed by group of INP in 1994 [7]. An electron beam moves over a periodic target and interacts with synchronism with electron beam surface wave. Surface waves are confined near the target surface at distance less than $\beta\gamma\lambda/(4\pi)$. It was performed a numerical simulation of such a FEL in [19], [21].

As it was mentioned above, VDFB could significantly enhance lasing process. For example VDFB can reduce interaction length and provide mode discrimination in oversized generating systems. Appearance of some coupled waves suppresses parasitic modes in the system. If only one mode is in synchronism with the electron beam, the threshold current j_{th} is proportional to the following value [8] :

$$j_{th} \sim \frac{1}{(kL)^3},$$

where $k = \omega/c$. If two modes are in synchronism with electrons at once we have the following estimate:

$$j_{th} \sim \frac{1}{(kL)^5},$$

and in the common case if n modes are in synchronism with electrons:

$$j_{th} \sim \frac{1}{(kL)^{3+2(n-1)}}.$$

So, threshold current can be significantly decreased when modes are degenerated in multiwave diffraction geometry if $kL \gg 1$. On the other hand interaction length can be reduced at given current value. Synchronism of n modes with electrons corresponds to n-folded roots degeneration of dispersion equation.

In VFEL operation, the linear stage quickly transfers to the nonlinear one, at which most of the electron beam energy transfers to radiation. For this reason, a detailed analysis is required for the nonlinear stage of VFEL, the regimes of generation, amplification, regenerative amplification at the nonlinear saturation stage and for the influence exerted on these regimes by gradual variations in the VDFB geometry. Computations of this kind are necessary for experiment design, optimal geometry determination and result processing. The system of equations for all cases of VFEL is obtained from Maxwell equations in the slowly-varying envelope approximation. In the general n-wave case the solution has the following form:

$$\mathbf{E} = \sum_{i=1}^{n} E_i e^{i(\mathbf{k_i r} - \omega t)}.$$

Here E_i are complex-valued amplitudes of electromagnetic field.

Expression for current density in the right-hand side of Maxwell equations can be obtained by averaging over the initial phases of electrons. This method is well known and is widely applied to the calculation of TWT, BWT, FEL and other electronic devices.

The system of equations for the case of three-wave VFEL can be written in the following form:

$$\frac{\partial E_0}{\partial t} + \gamma_0 c \frac{\partial E_0}{\partial z} + 0.5 i\omega l E_0 - 0.5 i\omega \chi_1 E_1 - 0.5 i\omega \chi_2 E_2 =$$

$$2\pi j \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \left(\exp(-i\Theta(t,z,p)) + \exp(-i\Theta(t,z,-p))\right) dp, \qquad (1)$$

$$\frac{\partial E_1}{\partial t} + \gamma_1 c \frac{\partial E_1}{\partial z} - 0.5 i\omega \chi_{-1} E_0 + 0.5 i\omega l_1 E_1 - 0.5 i\omega \chi_{2-1} E_2 = 0, \quad (2)$$

$$\frac{\partial E_2}{\partial t} + \gamma_2 c \frac{\partial E_2}{\partial z} - 0.5 i\omega \chi_{-2} E_0 - 0.5 i\omega \chi_{1-2} E_1 + 0.5 i\omega l_2 E_2 = 0, \quad (3)$$

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k_z - \frac{d\Theta(t,z,p)}{dz}\right)^3 \mathrm{Re}\left(E_0(t - z/u, z)\exp(i\Theta(t,z,p))\right), \quad (4)$$

$$\frac{d\Theta(t,0,p)}{dz} = k_z - \omega/u, \quad \Theta(t,0,p) = p,$$

$$E_0|_{z=0} = E_0^0, \quad E_1|_{z=L_1} = E_1^0, \quad E_2|_{z=L_2} = E_2^0, \quad (5)$$

$$E_m|_{t=0} = 0, \quad m = 0,1,2, \quad (6)$$

$$t > 0, \quad z \in [0, L], \quad p \in [-2\pi, 2\pi],$$

where values of boundaries L_2 and L_3 for wave vectors $\mathbf{k}_2$ and $\mathbf{k}_3$ take values 0 or L depending on geometry considered. $\Theta(t,z,p)$ describes the phase of electron beam relative to the electromagnetic wave, γ is Lorentz-factor of a beam. e, m are charge and mass of an electron respectively. γ_j are diffraction cosines, $\beta_{1,2} = \gamma_0/\gamma_{1,2}$ is diffraction asymmetry factors.

$$l = l_0 + \delta,$$

δ is a detuning from the Cherenkov condition. l_0, l_1, l_2 are system parameters corresponding to three waves:

$$l_i = \frac{\mathbf{k}_i^2 c^2 - \omega^2 \varepsilon_0}{\omega^2}.$$

ε_0 is a mean dielectric susceptibility and $\chi_{\pm j}$ are Fourier components of the dielectric susceptibility of the target.

$$\Phi = \sqrt{l_0 + \chi_0 - 1/(\beta\gamma)^2}.$$

System for n-wave VFEL looks like this:

$$\frac{\partial \mathbf{E}}{\partial t} + \sum_{i=1}^{N} \mathbf{A}_i \frac{\partial \mathbf{E}}{\partial x_i} + \mathbf{C}\mathbf{E} = \mathbf{F}(j), \quad (7)$$

where $\mathbf{E}$ is the vector of dimension M of amplitudes of electric field strength inside the target, j is the beam current density.

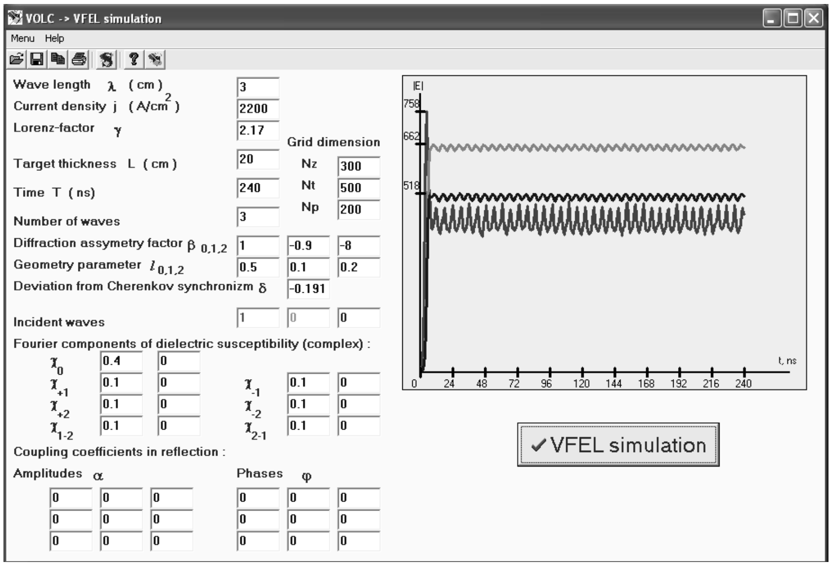


Figure 6: Interface of computer code VOLC

3 Results of numerical experiments

Numerical methods for simulation were developed and used effectively for different types of VFEL [24]-[26]. It allows to use parallel processing effectively. All these schemes proved itself be stable and converge to the solution of initial differential system. On their basis computer code VOLC that means *VOLume Code* for computer simulation of Volume FEL in two and three-wave geometries was created. Its interface is depicted in Fig.6.

We obtained numerous curves of electromagnetic amplitudes behavior: steady state solutions, limit cycle solutions and different chaotic regimes. E.g., in Fig.6 one can see limit-cycle regime of the triply periodic behavior of radiation pulse. In the next Fig.7 the periodic regime of VFEL intensity in three-wave geometry is given. It is evident that asymptotically we deal with periodic 2T regimes. This is confirmed by plots in Fourier space in Fig.8. In Fig.9 and Fig.10 one can see an example of chaotic regime of VFEL operation, corresponding phase space portrait and intensity in Fourier space.

To model an experiment, we have simulated the case of diffraction radiation which arises in the interaction of the exciting grating from the resonator with parameter $l_0 = 3$, corresponding to conventional BWO regime of VFEL-250. In Fig.11 dependence of VFEL intensity on detuning

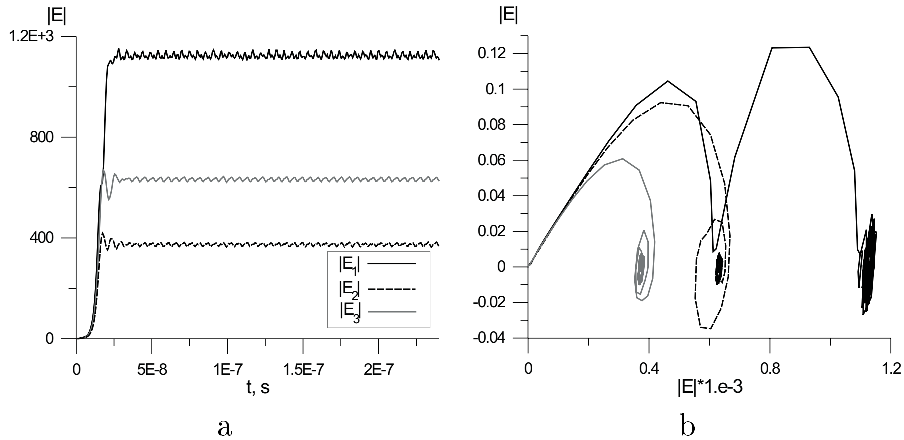


Figure 7: Periodic regimes of VFEL intensity (a) and corresponding phase space portrait (b)

from exact Cherenkov condition δ is shown. It is obvious the presence of domain with and without amplification. Domain with strong amplification corresponds to region of diffraction radiation.

Dependence of current threshold on length of the target for two-wave (solid curve) and three-wave geometry (dashed curve) is proposed in Fig.11b. This is a good illustration of effectiveness of volume distributed feedback. It is evident that the threshold current can be significantly decreased when modes are degenerated in multiwave diffraction geometry.

Three-wave geometry allows to widen the domain between first and second threshold points in generator for providing the steady regenerative amplification regime. Otherwise such regime is nonstable and goes to the regime of generation. Information contained in amplified signal will be lost. If beam current reaches first threshold point when radiation gain exceeds radiation loss, but at the same time is less than second threshold point of generation, the regenerative amplification takes place. After current reaches the second threshold point oscillation develops. Lines A and B correspond to the first and second threshold points respectively. In Fig.12 one can see three-wave geometry results. The distance between A and B is equal to 50 units, while in two-wave geometry it was a few units and only accounting of heavy absorption led it be extended till twenty units.

Demonstration of cases, when one mode in synchronism, two-root and three-root degeneration cases are adduced in [28]-[29].

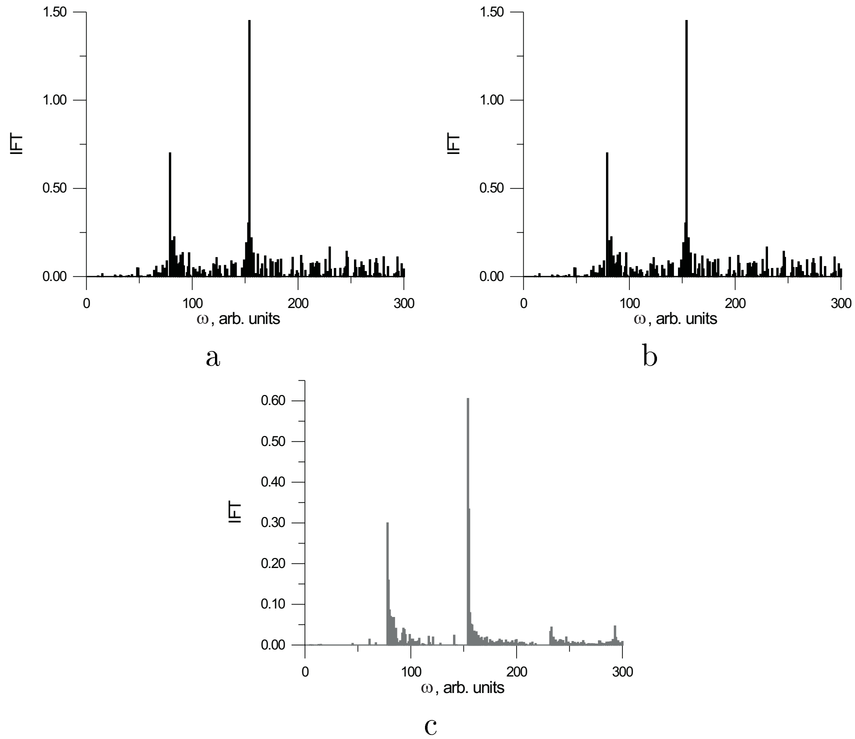


Figure 8: Fourier 2T periodic regime corresponding to amplitudes E_0 (a), E_1 (b), E_2 (c) from Fig.6

4 Conclusions

All results obtained numerically are in good agreement with analytical predictions. Next steps in VFEL simulation are simulation of experiment in INP with some diffracted gratings and analysis of different chaos regimes in VFEL.

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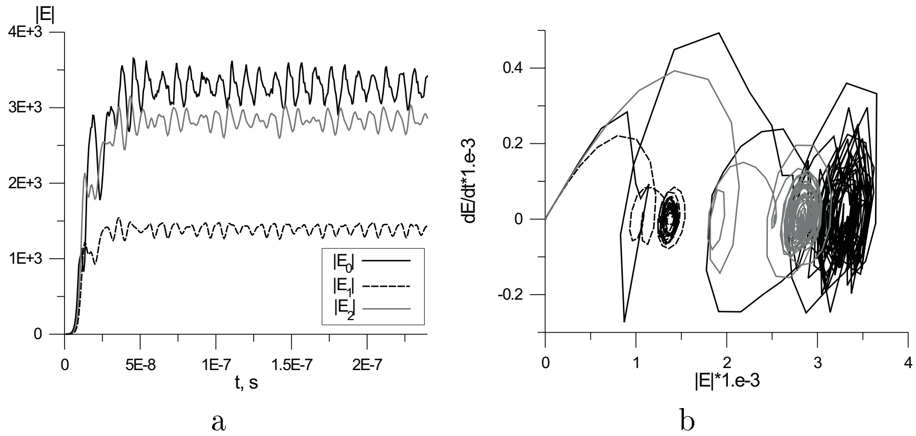

Figure 9: Chaotic regimes of VFEL intensity (a) and corresponding phase space portrait (b)

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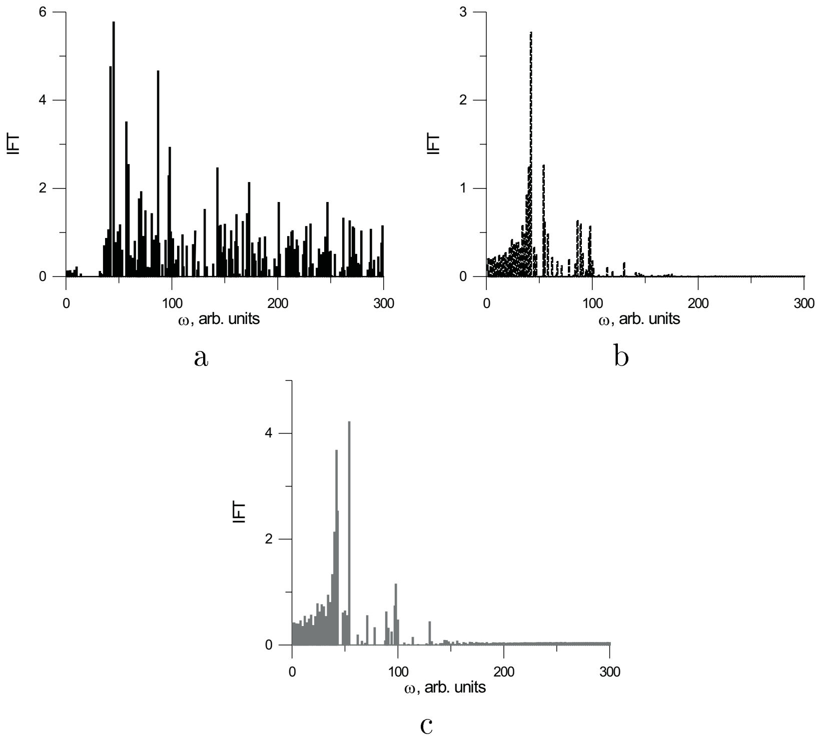

Figure 10: Fourier chaotic regime corresponding to amplitudes E_0 (a), E_1 (b), E_2 (c) from Fig.9

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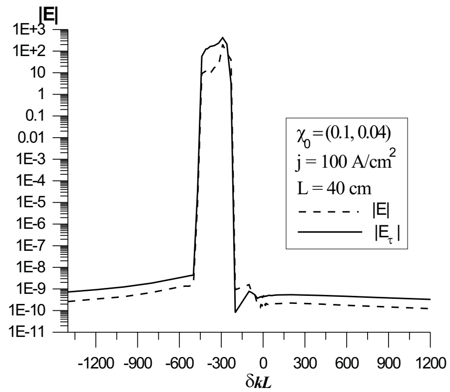

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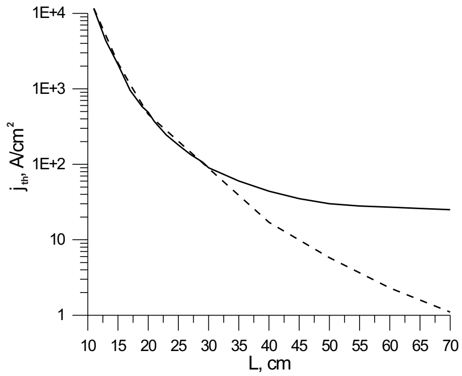

b

Figure 11: Diffraction radiation (a) and dependance of current threshold on lenght (b)

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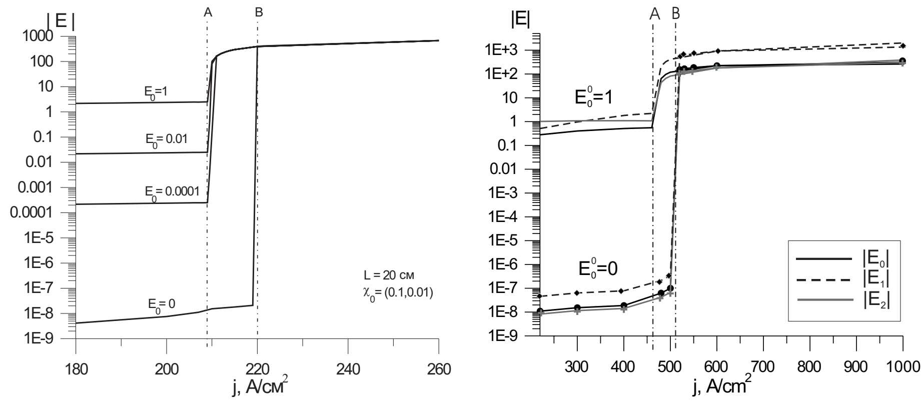

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Figure 12: Amplification and oscillation regimes in three- (a) and two-wave (b) VFEL

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